Supplementary Information

Investing in cooperation

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Overview

We study cooperation investments in three steps. Section 1 introduces the baseline model, a repeated trust game between a series of choosers (potential trust-givers) and an actor (potential reciprocator). We extend the model by allowing the actor to invest in cooperation in section 2 (investment model), incurring a cost to permanently reduce the cost of reciprocating choosers' trust. In section 3 (observable investment model), we make investment decisions observable to choosers, allowing them to condition trust on whether the actor initially invested.
We show that the domain of cooperation is extended in each step. In contrast to the main text, our analysis is general and applies to any distribution of actor types and a wider set of parameters. After deriving general results (propositions), we explain how they apply to the particular distributions considered in the main text (corollaries). We also characterize non-cooperative equilibria. Detailed proofs for each extension can be found in appendices A-C.
To complement our general mathematical analysis, we run simulations in a simple, tractable parameter case. Sections 4 and 5 provide strategy encodings and payoff derivations for the investment model and observable investment model simulations.

1 Baseline model

We introduce a repeated trust game between a series of choosers (trust-givers) and an actor (trust-reciprocator). We establish the conditions under which cooperation between them can emerge when costs are fixed by characterizing the unique cooperative equilibrium. In particular, we show that cooperation is difficult to sustain when the cost of reciprocation is high.

1.1 Model set up

1.1.1 General structure of the model

We consider an infinitely repeated trust game between one long-run actor, who plays all rounds of the game, and a series of short-run choosers, who each play one round only (adapted from Lie-Panis and André, 2022; building on the general framework of Mailath and Samuelson, 2006). To avoid the repetitive use of role names, we assign genders randomly: we use feminine pronouns (she/her) to refer to the actor, and masculine pronouns (he/him/his) to refer to choosers.

In each round $t \geq 1$, the actor is paired with a chooser, and both players play a trust game, as shown in Figure 1. The chooser first decides whether to trust or distrust the actor. Trust entails incurring a cost $\gamma > 0$ for the actor to gain a benefit $b > 0$. If trusted, the actor then decides whether to reciprocate or cheat. Reciprocation incurs cost $c_H > 0$ and yields benefit $\beta > 0$ for the chooser.

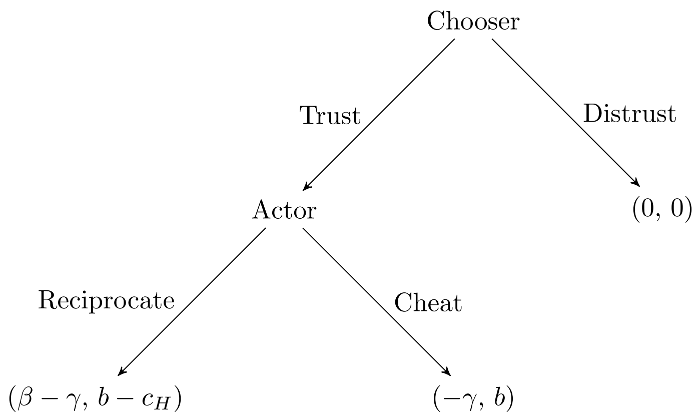
Note that we denote the cost of reciprocation as c_H, to indicate a potentially high cost relative to the benefit of being trusted b. This notation anticipates our later extensions, where we will allow the actor to invest in reducing this cost.

1.1.2 Reputation of the actor

Choosers observe only the actor's most recent action, which determines her **reputation**. This assumption simplifies the model by focusing on the most immediate and relevant information. Specifically:

- The actor has an **unknown reputation** if she has never been trusted by any chooser.
- The actor has a **good reputation** if, the last time she was trusted by a chooser, she reciprocated.
- The actor has a **bad reputation** if, the last time she was trusted by a chooser, she cheated.

Choosers can use this information to decide whether to trust the actor. The set of possible reputations is denoted $\Omega \equiv \{\text{unknown}, \text{good}, \text{bad}\}$. A pure strategy for choosers, represented by $\sigma_{\text{ch}} : \Omega \to \{\text{trust}, \text{distrust}\}$, specifies whether or not to trust the actor based on her reputation.



Supplementary Figure 1: Stage game for the baseline model. Payoffs are denoted as (chooser's payoff, actor's payoff).

1.1.3 Time preferences of the actor

The actor is characterized by her discount factor $\delta \in (0,1)$, which is drawn before the game begins according to a distribution function p. The support of p, denoted by $\Delta \subseteq (0,1)$, is the set of discount factors for which p is positive. The distribution can be discrete (e.g., the binary case with support $\{\delta_P, \delta_F\}$ considered in the main text), continuous (e.g., a truncated normal distribution on the entire interval $(0,1)$), or mixed.

For any subset $S \subseteq \Delta$, we denote by $\mathbb{P}(S) \equiv \int_S p(\delta)d\delta$ the probability that the actor's discount factor belongs to S. In the discrete case, this simplifies to $\mathbb{P}(S) = \sum_{\delta \in S} p(\delta)$. We refer to any subset S of Δ of positive measure (such that $\mathbb{P}(S) > 0$) as a set of **possible** discount factors.

The actor privately observes her discount factor δ before the game begins. Choosers do not observe δ, but they know the distribution p. We denote by

$$\mu \equiv \big(\, \mu(\delta \,|\, \text{unknown}),\, \mu(\delta \,|\, \text{good}),\, \mu(\delta \,|\, \text{bad}) \,\big)$$

the posterior beliefs of choosers about the actor's discount factor, conditional on her reputation. These posterior beliefs are shared by choosers and defined only over the support Δ of the distribution p. This ensures that choosers' expectations about the actor's behavior are based only on values that can actually occur with positive probability.

An actor strategy, represented by a function $\sigma_{\text{ac}} : \Delta \times \Omega \to \{\text{reciprocate}, \text{cheat}\}$, specifies whether to reciprocate a chooser's trust based on the actor's discount factor and current reputation.

1.1.4 Payoffs

We limit our analysis to pure strategy profiles $\sigma \equiv (\sigma_{\text{ch}}, \sigma_{\text{ac}})$. By assumption, when faced with an actor of reputation ω, a chooser who distrusts the actor gains null payoff $u_{\text{ch}}(\text{distrust} \mid \omega) \equiv 0$, where we denote the chooser's payoff as a function of his chosen action given the actor's reputation. A chooser who trusts in such a scenario pays $-\gamma$ and receives β if the actor then reciprocates; his payoffs are given by:

$$u_{\text{ch}}(\text{trust} \mid \omega) \equiv -\gamma + \mathbb{P}(\text{reciprocate} \mid \omega) \times \beta,$$

where the probability $\mathbb{P}(\text{reciprocate} \mid \omega)$ that the actor will reciprocate given that she has reputation ω depends on her strategy σ_{ac}, and therefore on the distribution of discount factors p. This probability is evaluated based on choosers' posterior beliefs $\mu(\cdot \mid \omega)$ — if for instance choosers believe that actors that have reputation ω have a low discount factor, and if these impatient actors always defect, then this probability is equal to 0. We omit σ_{ac} and $\mu(\cdot \mid \omega)$ from the notation for concision.

In contrast to choosers, the actor gains payoffs throughout the game, that she discounts according to her discount factor. When individuals play according to a strategy profile σ, the payoff of the actor given a discount factor δ is given by:

$$U_{\text{ac}}(\sigma \mid \delta) = (1-\delta) \sum_{t=1}^{\infty} \delta^t \pi_{\text{ac}}^t,$$

where π_{ac}^t is the payoff obtained by the actor in the trust game for round t, depending on the actions induced by σ — $\pi_{\text{ac}}^t = 0$ if the actor is not trusted in round t (e.g., because σ prescribed cheating in a previous round for this δ and distrusting given bad reputation), $\pi_{\text{ac}}^t = b - c_H$ if the actor is trusted and reciprocates, and $\pi_{\text{ac}}^t = b$ if she cheats.

1.1.5 Objective

To analyze our model, we study its Perfect Bayesian Equilibria, or PBEs. Following Fudenberg and Tirole (1991, Chapter 8), a PBE is a pair (σ, μ), where σ is a strategy profile and μ a system of beliefs, such that players have no profitable deviations in any possible scenario (on or off the equilibrium path), and beliefs are updated according to Bayes' rule whenever possible.

This requires considering all potential deviations. For the actor, we must verify that no type $\delta \in \Delta$ with any reputation ω has an incentive to deviate—even for reputation-type combinations that never occur on the equilibrium path (for example, a type who always cheats but somehow holds good reputation, or a type with good reputation when choosers always distrust initially, preventing this reputation from ever being reached). For choosers, we must consider deviations given every possible reputation, including those that never arise (for example, good reputation when every actor type cheats or when choosers never give initial trust).

We define a reputation ω as **possible** if there exists a set of possible discount factors $\mathcal{S} \subseteq \Delta$ and a sequence of past chooser actions (not necessarily prescribed by σ) under which any actor with discount factor $\delta \in \mathcal{S}$ would acquire reputation ω by following the strategy profile. Choosers' beliefs about types must be derived from Bayes' rule for all possible reputations, and may be specified arbitrarily otherwise.

This definition ensures that choosers form meaningful beliefs as widely as possible. Unknown reputation is always possible, since all types begin with it. Bad reputation is possible in any PBE (because there are always would-be cheaters), and good reputation is possible in any cooperative PBE, as defined below.

We concentrate our analysis on PBEs in which cooperation is possible. More precisely, a PBE is said to be **cooperative** if there exists a possible set of discount factors such that, whenever the actor's discount factor is drawn in this set, there exists a round $t \geq 1$ such that, along the outcome path of the repeated game, the chooser for round t trusts and the actor subsequently cooperates. For simplicity of notations, we assume that the actor reciprocates when indifferent with cheating, and that choosers trust when indifferent with distrusting. As we show below, these indifference cases occur only at knife-edge parameter values.

1.2 Cooperative equilibrium

1.2.1 General result

We show the existence of a unique cooperative PBE.

Proposition 1.1: Cooperative equilibrium

There is only one cooperative Perfect Bayesian Equilibrium. In this equilibrium, choosers trust the actor when she has an unknown or good reputation but distrust her when she has a bad reputation. Whatever her reputation, the actor reciprocates if and only if her discount factor δ satisfies:

$$\delta \geq \delta_0^{\text{recip.}} \equiv \frac{c_H}{b}. \tag{1.1}$$

The equilibrium exists if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) < 1. \tag{1.2}$$

We prove this proposition in Appendix A. The proof has three main steps. First, we establish that there can only be one chooser strategy in a cooperative equilibrium: choosers must initially trust the actor given unknown reputation for cooperation to occur, and they must discriminate between good and bad reputation to incentivize reciprocation. Second, we show that the actor reciprocates if and only if she is sufficiently patient, as captured by condition (1.1). This reflects her intertemporal trade-off: paying cost c_H to reciprocate a chooser's trust maintains her good reputation, ensuring future trust and the associated benefit b. Finally, we derive choosers' posteriors and the conditions under which this strategy profile constitutes a PBE. The left inequality in condition (1.2) ensures that choosers find it advantageous to trust initially, given that the actor has unknown reputation: the probability that the actor will reciprocate, $\mathbb{P}(\delta \geq \delta_0^{\text{recip.}})$, must exceed the cost-to-benefit ratio of trust, $\frac{\gamma}{\beta}$. The right inequality ensures that some actor types will cheat with positive probability, making it advantageous to distrust an actor with bad reputation. Otherwise, if all types reciprocated, choosers would stand to gain from deviation to trusting given bad reputation, whatever their beliefs.

1.2.2 Important special cases

We deduce conditions (B.1-3) for cooperation with a binary distribution, as considered in the main text, and the conditions with a continuous distribution.

Corollary 1.1: Binary distribution

If the actor's discount factor is δ_F with probability p and $\delta_P < \delta_F$ with probability $1 - p$, where $0 < p < 1$, the cooperative equilibrium exists if and only if:

$$c_H \leq \delta_F b, \tag{B.1}$$
$$c_H > \delta_P b, \tag{B.2}$$
$$\gamma \leq p\beta. \tag{B.3}$$

Corollary 1.2: Continuous distribution

If the actor's discount factor is continuously distributed over the entire interval $(0, 1)$, the cooperative equilibrium exists if and only if:

$$\gamma \leq \mathbb{P}(\delta \geq \delta_0^{\text{recip.}})\beta.$$

In particular, we see that there can be no cooperative equilibrium in the binary case if the cost of reciprocation makes reciprocation costly even for the future-oriented type ($c_H > \delta_F b$) — this is the case we focus on in the main analysis, creating a setting where cooperation investments become necessary.

With a continuous distribution, cooperation remains possible for higher costs — we only need $c_H < b$, to ensure the existence of reciprocating types ($\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) > 0$), and we also need these types to be sufficiently numerous so as to enable trust given unknown reputation ($\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) > \frac{\gamma}{\beta}$). For example, taking our default parameters used for Figures 2-4 of the main text ($b = 1$, $c_H = 0.9$, $\beta = 1$), and a normal distribution of mode $\mu = 0.5$ and standard deviation $\sigma = 0.25$ instead of the binary distribution with $p = 0.5$, $\delta_P = 0.25$ and $\delta_F = 0.75$ used for Figures 2 and 3, we obtain $\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) \approx 0.0336$ — the cooperative equilibrium is then possible only for very small values of the cost of trust γ.

1.3 Other non-cooperative equilibria

We demonstrate the existence of only two other PBEs, detailed in Appendix A, neither of which is cooperative. In the trivial equilibrium, choosers always distrust the actor, and the actor never reciprocates; this equilibrium exists for all parameter values. In the non-cooperative equilibrium, choosers trust the actor only given good reputation, and the actor reciprocates if and only if her discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$, as in the cooperative equilibrium; this equilibrium exists if and only if $0 < \mathbb{P}(\delta \geq \frac{c_H}{b}) \leq \frac{\gamma}{\beta}$.

Thus, cooperation can fail for one of two reasons: because choosers do not incentivize reciprocation (trivial equilibrium), or because even though they do, by trusting given good reputation and distrusting given bad reputation, the fraction of reciprocating types is insufficient to justify initial trust, given unknown reputation (non-cooperative equilibrium).

2 Investment model

We extend the baseline model by allowing the actor to invest in reducing the cost of reciprocation before interactions with choosers begin. We obtain a unique cooperative equilibrium. Investment facilitates cooperation: we show that this equilibrium occurs under a wider parameter domain, including cases where the baseline cost of reciprocation is high.

2.1 Changes to the baseline model

Building on the baseline model introduced in Section 1, we now allow the actor to invest in cooperation with choosers. To do so, we add a round before the repeated trust game. Initially, in round 0, the actor decides whether to invest. Investing incurs cost k but permanently lowers the cost of reciprocation from c_H to c_L, where $0 < c_L \leq c_H$ and $c_L < b$. Subsequently, for every round $t \geq 1$, the actor plays the repeated trust game, the only difference with the baseline model being that her cost of reciprocation is lower if she initially invested.

An actor strategy $\sigma_{\text{ac}} \equiv (\sigma_0, \sigma_t)$ is now represented by a pair of functions. The first, $\sigma_0 : \Delta \to \{\text{invest}, \text{do not invest}\}$, specifies whether to initially invest based on the actor's discount factor; the second, $\sigma_t : \Delta \times \{\text{invested}, \text{didn't invest}\} \times \Omega \to \{\text{reciprocate}, \text{cheat}\}$, specifies whether to reciprocate a chooser's trust based on the actor's discount factor, whether she initially invested and her reputation.

We make no other changes to the baseline model. In particular, in this extension, the actor's investment decision is not observed by choosers. As a result, reputation dynamics are unchanged: the actor begins the repeated trust game with unknown reputation. Her reputation changes every time she is trusted to reflect her last reciprocation decision; she acquires good reputation every time she reciprocates a chooser's trust, and bad reputation every time she cheats.

For simplicity of notations, we employ similar tie-breaking assumptions as before, assuming that the actor invests whenever indifferent between investing and not investing.

2.2 Cooperative equilibrium

2.2.1 General result

We show the existence of a unique cooperative PBE.

Proposition 2.1: Cooperative equilibrium

There is only one cooperative Perfect Bayesian Equilibrium. In this equilibrium, choosers trust the actor when she has an unknown or good reputation but distrust her when she has a bad reputation. The actor initially invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_1^{\text{invest}} \equiv \max\{\frac{c_L - k + \sqrt{(k - c_L)^2 + 4kb}}{2b}, \frac{k}{k + c_H - c_L}\}. \tag{2.1}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$; after not investing, she reciprocates if and only if $\delta \geq \frac{c_H}{b}$. Along the outcome path, the actor reciprocates if and only if:

$$\delta \geq \delta_1^{\text{recip.}} \equiv \min\{\frac{c_L - k + \sqrt{(k - c_L)^2 + 4kb}}{2b}, \frac{c_H}{b}\}. \tag{2.2}$$

This equilibrium exists if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_1^{\text{recip.}}) < 1. \tag{2.3}$$

We prove this proposition in Appendix B. The proof builds on the proof for the baseline model, showing that choosers must adopt the same strategy, and that the actor will reciprocate based on her discount factor and the cost of reciprocation she faces. This leads to (at most) three types of behavior in trust games: the actor always reciprocates if she is patient ($\delta \geq \frac{c_H}{b}$), always cheats if she is impatient

($\delta < \frac{c_L}{b}$), and when her patience is intermediate ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$), she reciprocates if she initially invested but cheats if she initially opted out of investing.

Knowing her behavior in every possible trust game, we then analyze her investment decision, obtaining condition (2.1). Putting everything together, we deduce her reciprocation behavior along the outcome path, yielding condition (2.2). Note that there are two cases for behavior along the outcome path. When $\frac{k}{k+c_H-c_L} \leq \frac{c_H}{b}$, both thresholds are equal ($\delta_1^{\text{invest}} = \delta_1^{\text{recip.}}$). The actor both invests and reciprocates if she is sufficiently patient, and otherwise opts out of investing and subsequently cheats. When in contrast $\frac{k}{k+c_H-c_L} > \frac{c_H}{b}$, the threshold for investing is higher ($\delta_1^{\text{invest}} > \delta_1^{\text{recip.}}$). The actor invests and reciprocates if patient ($\delta \geq \frac{k}{k+c_H-c_L}$), opts out and cheats if impatient ($\delta < \frac{c_H}{b}$), and with intermediate patience ($\frac{c_H}{b} \leq \delta < \frac{k}{k+c_H-c_L}$), she opts out of investing but nevertheless reciprocates choosers' trust.

Finally, having established the actor's behavior in every possible case, we derive choosers' posteriors and condition (2.3). This condition is similar to the one obtained for the baseline model, condition (1.2), because we have the same three reputational states, with choosers having to trust given good and unknown reputation and distrust given bad reputation. The only difference is the threshold, $\delta_1^{\text{recip.}}$ replacing $\delta_0^{\text{recip.}}$.

2.2.2 Extension of the cooperative domain

We obtain cooperation under a wider parameter domain than in the baseline model if and only if:

$$\mathbb{P}(\delta \geq \delta_1^{\text{recip.}}) > \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}).$$

In other words, investment extends the domain of cooperation whenever $\delta_1^{\text{recip.}} < \delta_0^{\text{recip.}}$ and the set of discount factors $(\delta_1^{\text{recip.}}, \delta_0^{\text{recip.}})$ is possible, enabling cooperation for higher values of the cost-to-benefit ratio of trust $\frac{\gamma}{\beta}$.

We show that:

$$\delta_0^{\text{recip.}} > \delta_1^{\text{recip.}} \iff \frac{c_H}{b} > \frac{k}{k + c_H - c_L},$$

which, re-arranging, equivalently leads:

$$c_H(c_H - c_L) > k(b - c_H).$$

Thus, we obtain a lower threshold for reciprocation if the baseline cost of cooperation was prohibitive ($c_H \geq b$), in which case the right member of this inequality is negative or null, and also, when $c_H < b$, if the cost of investing is sufficiently small; i.e., if $k < c_H(c_H - c_L)/(b - c_H)$. For example, taking our default parameters, this inequality becomes $k < 3.6$; over the entire space considered in Figures 2-4, investment lowers the threshold for reciprocation.

2.2.3 Important special cases

We deduce the conditions for cooperation with a binary distribution and a continuous one.

Corollary 2.1: Binary distribution

If the actor's discount factor is δ_F with probability p and $\delta_P < \delta_F$ with probability $1 - p$, where $0 < p < 1$, the cooperative equilibrium exists if and only if:

$$\delta_F \geq \delta_1^{\text{recip}},$$
$$\delta_P < \delta_1^{\text{recip}},$$
$$\gamma \leq p\beta.$$

In particular, when in addition $c_H > \delta_F b \geq c_L$ and $c_L > \delta_P b$, the present-oriented type opts out of investing and cheats, whereas the future-oriented type reciprocates along the outcome path if and only if she finds it initially advantageous to invest. The cooperative equilibrium exists if and only if:

$$\delta_F \geq \frac{c_L - k + \sqrt{(k - c_L)^2 + 4kb}}{2b} \iff k \leq \frac{\delta_F}{1 - \delta_F}(\delta_F b - c_L), \tag{I.1}$$

$$\gamma \leq p\beta. \tag{I.2}$$

Corollary 2.2: Continuous distribution

If the actor's discount factor is continuously distributed over the entire interval $(0, 1)$, the cooperative equilibrium exists if and only if:

$$\gamma \leq \mathbb{P}(\delta \geq \delta_1^{\text{recip.}})\beta.$$

3 Observable investment model

We now allow choosers to observe the actor's investment decision, further extending our model. We obtain two types of cooperative equilibria. In signaling equilibria, choosers discriminate based on investment behavior. Observation further facilitates cooperation: we show that these equilibria occur under a wider domain than the cooperative equilibrium of the previous extension. We also characterize the second type of equilibrium, in which choosers trust regardless of initial investment decision, which occurs under a limited parameter domain.

3.1 Changes to the investment model

We now assume that choosers observe the actor's initial investment decision. In particular, choosers know the actor's cost of reciprocation in all subsequent rounds: cost c_L if she invested, and c_H if she didn't. Choosers continue to observe the actor's most recent trust game decision, as in the previous two models. The set of reputations is now equal to:

$$\Omega' \equiv \{\text{invested}, \text{didn't invest}\} \times \Omega,$$

reflecting the actor's initial investment decision and her last trust game decision, if any. For example, if the actor initially invests and subsequently reciprocates trust, she enters round 1 with reputation $(\text{invested}, \text{unknown})$, and round 2 with reputation $(\text{invested}, \text{good})$.

We make no other changes to the model. A chooser strategy σ_{ch} specifies whether to trust the actor depending on her reputation, and is defined over the extended reputational set Ω'. Choosers' posterior beliefs μ now comprise six functions; one per reputation. An actor strategy σ_{ac} is defined as in the previous extension, specifying reciprocation decision over the extended reputational set.

3.2 Signaling equilibria

We show the existence of two types of cooperative PBEs. In the first type, which we call signaling equilibria (detailed below), choosers discriminate based on investment behavior, giving the actor an additional incentive to invest. Within this first type, we identify two distinct signaling equilibria that differ in how choosers treat actors who didn't invest but subsequently acquired good reputation. In the second type, which we call the universal trust equilibrium (detailed in section 3.3), choosers trust the actor both after she invested and after she didn't invest.

3.2.1 General results

We show the existence of two signaling equilibria.

Proposition 3.1: Signaling equilibria

In any signaling equilibrium, choosers trust the actor when her reputation is (invested, unknown) or (invested, good), but distrust her when her reputation is (invested, bad), (didn't invest, unknown), or (didn't invest, bad). Their action for reputation (didn't invest, good) depends on the specific equilibrium. The actor initially invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_{2,s}^{\text{invest}} \equiv \min\{\frac{k}{b}, \frac{k}{k+b-c_L}\}. \tag{3.1}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$, whatever her reputation. Her behavior after not investing depends on the specific equilibrium. Along the outcome path, the actor reciprocates if and only if:

$$\delta \geq \delta_{2,s}^{\text{recip.}} \equiv \max\{\frac{c_L}{b}, \frac{k}{k+b-c_L}\}. \tag{3.2}$$

Proposition 3.2: First signaling equilibrium

In the first signaling equilibrium, choosers distrust the actor when her reputation is (didn't invest, good). After not investing, the actor always cheats.
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}}), \tag{3.3}$$
$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.4}$$
$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest.}}). \tag{3.5}$$

Proposition 3.3: Second signaling equilibrium

In the second signaling equilibrium, choosers trust the actor when her reputation is (didn't invest, good). After not investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_H}{b}$.
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}}), \tag{3.3}$$
$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.4}$$
$$0 < \mathbb{P}(\delta \geq \frac{c_H}{b}), \tag{3.6}$$
$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}). \tag{3.5}$$
$$\delta_{2,s}^{\text{invest}} \leq \frac{c_H}{b} \text{ or } \mathbb{P}(\delta \geq \frac{c_H}{b} \mid \delta < \delta_{2,s}^{\text{invest}}) < \frac{\gamma}{\beta}. \tag{3.7}$$

We prove these propositions in Appendix C. As with other proofs, we begin by noting that choosers' actions are constrained: they must discriminate based on initial investment behavior to obtain a signaling equilibrium, and they must discriminate between good and bad reputation along the outcome path, to incentivize actor reciprocation. The first and second signaling equilibrium differ in how choosers treat the actor when her reputation is (didn't invest, good), a reputation which does not occur along the outcome path.

We obtain the actor's strategy in every possible trust game, and deduce her behavior along the outcome path, obtaining conditions (3.1) and (3.2). There are two cases. When $k \geq c_L$, both thresholds are equal ($\delta_{2,s}^{\text{invest}} = \delta_{2,s}^{\text{recip.}}$), and investment is a perfect signal of reciprocation: the actor initially invests and subsequently reciprocates trust if patient, and otherwise does neither. When in contrast $k < c_L$, investment is easier than reciprocation ($\delta_{2,s}^{\text{invest}} < \delta_{2,s}^{\text{recip.}}$). The actor invests and reciprocates if patient ($\delta \geq \frac{c_L}{b}$), does neither if impatient ($\delta < \frac{k}{b}$), and invests but cheats if of intermediate patience ($\frac{k}{b} \leq \delta < \frac{c_L}{b}$). Proposition 3.1 applies to both signaling equilibria.

Proposition 3.2 applies to the first signaling equilibrium, in which choosers always distrust non-investors, and the actor always cheats after opting out of investing. Condition (3.3) guarantees the existence of reciprocator types, ensuring the equilibrium is cooperative, while condition (3.4) guarantees the existence of types who would cheat after investing, making it sensible to distrust given $(\text{invested}, \text{bad})$. Condition (3.5) guarantees that investment is a sufficiently good predictor of reciprocation, enabling trust for investors.

Proposition 3.3 applies to the second signaling equilibrium, in which choosers trust the actor when her reputation is $(\text{didn't invest}, \text{good})$, leading the actor to reciprocate after not investing if $\delta \geq \frac{c_H}{b}$. We obtain a PBE when the previous three conditions are satisfied, as well as two new ones: condition (3.4) guarantees the existence of types who would reciprocate after not investing, enabling trust given $(\text{didn't invest}, \text{good})$, while condition (3.5) makes it sensible to distrust non-investors, taking into account cases where investment is more difficult than incurring the high cost of reciprocation. As visible in Supplementary Figure 3, these cases require a high cost of investing so as to satisfy $k > \frac{c_H(b-c_L)}{b-c_H}$ — which yields $k > 4.5$ with our default parameters.

3.2.2 Extension of the cooperative domain

We obtain cooperation (in the first signaling equilibrium) under a wider parameter domain than in the first extension if and only if the sets of discount factors $(0, \delta_{2,s}^{\text{recip.}}]$ and $(0, \frac{c_L}{b})$ are possible, guaranteeing the existence of reciprocators and types that would cheat even after investing, and

$$\mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) > \mathbb{P}(\delta \geq \delta_1^{\text{recip.}}).$$

This condition is always satisfied. Cooperation is facilitated for two reasons: first, because choosers no longer need to trust blindly an actor who never interacted before, and can instead use the actor's initial investment behavior, and second, because the actor is more likely to reciprocate ($\delta_{2,s}^{\text{recip.}} < \delta_{1,s}^{\text{recip.}}$), since there is now an additional incentive to invest, to attract the trust of choosers, enabling reciprocation for more actor types.

3.2.3 Important special cases

We deduce the conditions for signaling equilibria with a binary distribution and a continuous one.

Corollary 3.1: Binary distribution - first signaling equilibrium

If the actor's discount factor is δ_F with probability p and $\delta_P < \delta_F$ with probability $1-p$, where $0 < p < 1$, the first signaling equilibrium exists if and only if:

$$\delta_F \geq \delta_{2,s}^{\text{recip}},$$
$$\delta_P < \frac{c_L}{b},$$
$$\gamma \leq \mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}})\beta.$$

Under these conditions, the future-oriented type invests and reciprocates, and the present-oriented type always cheats, including after investing. The present-oriented type initially invests if $k \leq \delta_P b$, in which case $\mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) = p$, and opts out of investing if $k > \delta_P b$, in which case $\mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) = 1$.
In particular, when we assume $c_H > \delta_F b \geq c_L$ and $c_L > \delta_P b$, as we do in the main text, we obtain the honest signaling equilibrium iff:

$$\delta_F \geq \frac{k}{k+b-c_L} \iff k \leq \frac{\delta_F}{1-\delta_F}(b-c_L), \tag{HS.1}$$
$$k > \delta_P b, \tag{HS.2}$$
$$\gamma \leq \beta, \tag{HS.3}$$

and the pooling equilibrium iff:

$$k \leq \delta_P b, \tag{P.1}$$
$$\gamma \leq p\beta. \tag{P.2}$$

Corollary 3.2: Binary distribution - second signaling equilibrium

If the actor's discount factor is δ_F with probability p and $\delta_P < \delta_F$ with probability $1-p$, where $0 < p < 1$, the second signaling equilibrium exists if and only if:

$$\begin{aligned} \delta_F &\geq \max\{\delta_{2,s}^{\text{recip}}, \frac{c_H}{b}\}, \\ \delta_P &< \frac{c_L}{b}, \\ \gamma &\leq \mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}})\beta. \end{aligned}$$

Under these conditions, the future-oriented type invests and reciprocates, and would also reciprocate after not investing, and the present-oriented type always cheats, including after investing. The present-oriented type initially invests if $k \leq \delta_P b$, in which case $\mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) = p$, and opts out of investing if $k > \delta_P b$, in which case $\mathbb{P}(\delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) = 1$.
In particular, the second signaling equilibrium does not exist when $c_H > \delta_F b$, as we assume in the main text.

Corollary 3.3: Continuous distribution

If the actor's discount factor is continuously distributed over the entire interval $(0, 1)$, the first signaling equilibrium exists if and only if:

$$\gamma \leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest.}})\beta.$$

The second signaling equilibrium exists if and only if:

$$\begin{aligned} c_H &< b, \\ \gamma &\leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest.}})\beta, \\ \delta_{2,s}^{\text{invest}} \leq \frac{c_H}{b} &\text{ or } \mathbb{P}(\delta \geq \frac{c_H}{b} \mid \delta < \delta_{2,s}^{\text{invest}})\beta < \gamma. \end{aligned}$$

3.3 Universal trust equilibrium

3.3.1 General result

We show the existence of a unique cooperative PBE in which choosers do not discriminate based on initial investment behavior.

Proposition 3.4: Universal trust

In the universal trust equilibrium, choosers trust the actor when her reputation is (invested, unknown), (invested, good), (didn't invest, unknown), or (didn't invest, good), and distrust her given (invested, bad) or (didn't invest, bad). The actor invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_{2,u}^{\text{invest}} \equiv \frac{k}{k + c_H - c_L}. \tag{3.8}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$; after not investing, she reciprocates if and only if $\delta \geq \frac{c_H}{b}$. Overall, the actor reciprocates if and only if her discount factor satisfies:

$$\delta \geq \delta_{2,u}^{\text{recip.}} \equiv \frac{c_H}{b}. \tag{3.9}$$

This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,u}^{\text{invest}}), \tag{3.10}$$

$$0 < \mathbb{P}(\delta_{2,u}^{\text{recip.}} \leq \delta < \delta_{2,u}^{\text{invest}}), \tag{3.11}$$

$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.12}$$

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,u}^{\text{recip.}} \mid \delta < \delta_{2,u}^{\text{invest}}). \tag{3.13}$$

We prove this proposition in Appendix C. To do so, we begin by noting that since choosers trust regardless of investment behavior by definition, the actor must reciprocate with positive probability after investing and not investing, which means that choosers must incentivize reciprocation in all subgames, defining their strategy. As a result, the actor faces the same incentive to invest and reciprocate as in the cooperative equilibrium of the first extension. We deduce conditions (3.8) and (3.9), which are degenerate versions of (2.1) and (2.2), by noting that there must exist types that do not invest but reciprocate, meaning that we must be in the case where $\frac{k}{k+c_H-c_L} > \frac{c_H}{b}$. Condition (3.10) guarantees the existence of types that invest and reciprocate, condition (3.6) the existence of types that do not invest but reciprocate, and condition (3.12) the existence of types who would cheat even after investing. Finally, condition (3.13) enables trust for non-investors.

3.3.2 Cases of interest

We deduce the conditions for universal trust with a binary distribution and a continuous one. In particular, we note that universal trust is impossible given a binary distribution, and that it requires $\frac{c_H}{b} < \frac{k}{k+c_H-c_L}$, which becomes $k > 3.6$ with our default parameters, given a continuous distribution. Over the entire space considered in Figures 2-4, universal trust is impossible.

Corollary 3.4: Binary distribution

If the actor's discount factor is either δ_F with probability p or $\delta_P < \delta_F$ with probability $1-p$, where $0 < p < 1$, the universal trust equilibrium does not exist. This equilibrium requires at least three types, as per conditions (3.10-3.12).

Corollary 3.5: Continuous distribution

If the actor's discount factor is continuously distributed over the entire interval $(0, 1)$, the universal trust equilibrium exists if and only if:

$$\delta_{2,u}^{\text{recip.}} < \delta_{2,u}^{\text{invest}},$$

$$\gamma \leq \mathbb{P}(\delta \geq \delta_{2,u}^{\text{recip.}} \mid \delta < \delta_{2,u}^{\text{invest}})\beta.$$

4 Material for simulations of the investment model

We describe the strategy encodings and derive payoffs for the simulation of the investment model.

To complement our general equilibrium analysis, we run evolutionary simulations of the investment model in the case of a binary distribution. We consider populations of n_A actors and n_C choosers, where each actor is future-oriented ($\delta = \delta_F$) with probability p and present-oriented ($\delta = \delta_P < \delta_F$) with probability $1 - p$.

We update strategies using a pairwise comparison process (Traulsen et al., 2006). To that end, we calculate the expected payoff of every strategy given the current population distributions, assuming $n_A \gg 1$ and $n_C \gg 1$. The implementation uses $n_A = n_C = 50$; all other parameters are specified in Methods.

Below we describe the strategy encodings and derive payoffs.

4.1 Strategy encodings

4.1.1 Actors

An actor strategy is a 4-bit vector

$$(I_F, R_F, I_P, R_P) \in \{0,1\}^4,$$

specifying whether to initially invest ($I_F = 1$) and later reciprocate trust ($R_F = 1$) if future-oriented, and whether to invest ($I_P = 1$) and reciprocate ($R_P = 1$) if present-oriented. This restricts attention to reputation-independent reciprocation (strategies σ_t that only depend on δ), which are always chosen in a PBE.

There are $2^4 = 16$ actor strategies, indexed by $i \in \{0, 1, \ldots, 15\}$. For example, following strategy $(0, 1, 1, 0)$, indexed by $2^2 + 2^1 = 6$, an actor does not invest but reciprocates trust if future-oriented, and invests and cheats if present-oriented. For every i, we denote by $x_i \in [0, 1]$ the fraction of actors using strategy i and represent the population of actors as $X = (x_i)_{i=0}^{15}$ with $\sum_i x_i = 1$.

4.1.2 Choosers

A chooser strategy is a 3-bit vector

$$(T_U, T_B, T_G) \in \{0,1\}^3,$$

specifying whether to trust actors with unknown ($T_U = 1$), bad ($T_B = 1$) or good ($T_G = 1$) reputation.

There are $2^3 = 8$ chooser strategies, indexed by $j \in \{0, 1, \ldots, 7\}$. For example, following strategy $(0, 1, 1)$, indexed by 3, a chooser trusts actors with good or bad reputation, and distrust those with unknown reputation. For every j, we denote by $y_j \in [0, 1]$ the fraction of choosers using strategy j and represent the population of choosers as $Y = (y_j)_{j=0}^{7}$ with $\sum_j y_j = 1$.

4.2 Derivation of actor payoffs

4.2.1 Probabilities of being trusted

To derive the payoff of every actor strategy $i \in \{0, 1, \ldots, 15\}$ given any distribution of chooser strategies Y, we begin by defining the probabilities θ_U, θ_B and θ_G that an actor is trusted when she holds unknown, bad or good reputation, respectively, which are equal to:

$$\theta_U = y_4 + y_5 + y_6 + y_7, \qquad \theta_B = y_2 + y_3 + y_6 + y_7, \qquad \theta_G = y_1 + y_3 + y_5 + y_7.$$

Using these probabilities, we then calculate the payoff of all four actor action pairs (I_X, R_X), given an unspecified discount factor $\delta_X \in \{\delta_P, \delta_F\}$. We index these four action pairs (0,0), (0,1), (1,0), (1,1), and their associated payoffs, with $i \in \{0, 1, 2, 3\}$, consistently with how we index actor strategies — for example, the action pair $(\text{invest}, \text{cheat})$ is indexed by $1 \times 2^1 + 0 \times 2^0 = 2$.

We assume $\theta_U > 0$ from here on, and obtain formulas that generalize to the case where $\theta_U = 0$. If $\theta_U = 0$, actors maintain unknown reputation throughout the game and are never trusted, earning lifetime payoff $-k\,(1 - \delta)$ if they initially invested and 0 if they did not.

4.2.2 Payoff of actors who don't invest and cheat

If an actor doesn't invest and cheats, she incurs no cost in round 0 and enters round 1 with unknown reputation. She subsequently keeps unknown reputation as long as she is not trusted. Once trusted, she acquires and maintains bad reputation throughout the rest of the game.

Since she is eventually trusted ($\theta_U > 0$), in the long-run, she obtains on average payoff

$$U^{\infty}_{\text{ac},0} = \theta_B\, b,$$

reflecting the fact that, in any given round, she is trusted by choosers with probability θ_B, earning b, and subsequently cheats, incurring no cost.

Starting from round 1, which she enters with unknown reputation, her payoff is:

$$U^{t\geq 1}_{\text{ac},0} = \theta_U\,((1-\delta)\,b + \delta\, U^{\infty}_{\text{ac},0}) + (1-\theta_U)\,(0 + \delta\, U^{t\geq 1}_{\text{ac},0}),$$

reflecting the fact that she receives b that round and subsequently acquires bad reputation, entering the long-term regime, if she is trusted, with probability θ_U, and otherwise maintains unknown reputation, keeping the transitory payoff $U^{t\geq 1}_{\text{ac},0}$.

Re-organizing, and using $U^{\infty}_{\text{ac},0} = \theta_B b$, we deduce:

$$U^{t\geq 1}_{\text{ac},0}\,(1 - (1-\theta_U)\delta) = \theta_U(1 - \delta + \delta\theta_B)b,$$

which is equivalent to:

$$U^{t\geq 1}_{\text{ac},0} = \frac{1-(1-\theta_B)\delta}{1-(1-\theta_U)\delta}\,\theta_U b.$$

Finally, since the actor does not invest in round 0, her lifetime payoff is

$$U_{\text{ac},0} = 0 + \delta\, U^{t\geq 1}_{\text{ac},0},$$

which yields:

$$U_{\text{ac},0}(\delta) = \frac{1-(1-\theta_B)\delta}{1-(1-\theta_U)\delta}\,\delta\theta_U b. \tag{4.1}$$

Note that this formula extends to the case where $\theta_U = 0$, in which case $U_{\text{ac},0} = 0$. (This is also the case for the three other action pairs.)

4.2.3 Payoff of actors who don't invest and reciprocate trust

Two things change for an actor who doesn't invest but reciprocates trust: each time she is trusted, she incurs cost c_H instead of no cost, and she obtains good reputation, which she subsequently maintains throughout rest of the game.

Her long-run payoff is equal to her payoff once in good reputation:

$$U^{\infty}_{\text{ac},1} = \theta_G\,(b - c_H).$$

Her payoff from round 1 is:

$$U^{t\geq 1}_{\text{ac},1} = \theta_U\,((1-\delta)\,(b - c_H) + \delta\, U^{\infty}_{\text{ac},1}) + (1-\theta_U)\,(0 + \delta\, U^{t\geq 1}_{\text{ac},1}),$$

since she exits the transitory unknown reputation for the absorbent good reputation when she is trusted, with probability θ_U, in which case she recieves $b - c_H$ that round.

Re-organizing as above, we deduce:

$$U^{t\geq 1}_{\text{ac},1} = \frac{1-(1-\theta_G)\delta}{1-(1-\theta_U)\delta}\,\theta_U(b - c_H).$$

Finally, her lifetime payoff is obtained by multiplying by δ as above:

$$U_{\text{ac},1}(\delta) = \frac{1-(1-\theta_G)\delta}{1-(1-\theta_U)\delta}\,\delta\theta_U(b - c_H). \tag{4.2}$$

4.2.4 Payoff of actors who invest and cheat

If an actor invests but cheats, only one things changes with respect to the first action pair: she incurs cost k in round 0, with no effect on her later payoff. Her lifetime payoff is obtained by subtracting $(1-\delta)k$ to the expression for $U_{\text{ac},0}$. We obtain:

$$U_{\text{ac},2}(\delta) = -(1-\delta)\,k + \frac{1-(1-\theta_B)\delta}{1-(1-\theta_U)\delta}\,\delta\theta_U b. \tag{4.3}$$

4.2.5 Payoff of actors who invest and reciprocate

If an actor invests and reciprocates trust, two things change with respect to the second action pair: she initially incurs cost k, and subsequently lower her cost of reciprocation from c_H to c_L. Her lifetime payoff is obtained by replacing c_H with c_L in the expression for $U_{\text{ac},1}$ and subtracting $(1-\delta)k$. We obtain:

$$U_{\text{ac},3}(\delta) = -(1-\delta)\,k + \frac{1-(1-\theta_G)\delta}{1-(1-\theta_U)\delta}\,\delta\theta_U(b-c_L). \tag{4.4}$$

4.2.6 Payoff of any given strategy

We use the expressions above to obtain the expected payoff of any actor strategy by mixing across both possible discount factors. For example, the payoff of strategy $(1,0,0,1)$ is

$$p\,U_{\text{ac},2}(\delta_F) + (1-p)\,U_{\text{ac},1}(\delta_P),$$

since the actor plays $(\text{invest}, \text{cheat})$ if future-oriented and $(\text{do not invest}, \text{reciprocate})$ if present-oriented.

More generally, for any strategy (I_F, R_F, I_P, R_P) the expected payoff is the weighted average of the two corresponding action-pair payoffs—namely, p times the payoff under (I_F, R_F) at δ_F plus $(1-p)$ times the payoff under (I_P, R_P) at δ_P.

4.3 Derivation of chooser payoffs

In our model, choosers are short-run players, who earn payoff $u_{\text{ch}}(\text{trust} \mid \omega)$ if they trust an actor that holds reputation ω. Choosers must make sensible decisions in every case—in any SPE, they cannot have beneficial deviations for any $\omega \in \Omega$. This leads us to calculate choosers' payoffs conditional on all three possible reputations.

For the evolutionary simulations, however, we need a single payoff metric that is not conditional on a particular reputation, so that we can compare one strategy's performance to another's and update accordingly. To that end, we replace our short-run choosers with long-run ones, who participate in all rounds and share a common discount factor δ.

This amounts to assigning a geometric distribution over rounds,

$$\mathbb{P}(T=t) = (1-\delta)\delta^{t-1},$$

so that earlier and later encounters are weighted consistently. Note that, as evidenced below, time t caries no information about reciprocation, and only affects the distribution of reputations.

Below, we describe the distribution of actor reputations across rounds, and derive payoffs for choosers who trust only actors with unknown, good, or bad reputation. Payoffs for any chooser strategy follow directly from these three cases.

4.3.1 Distribution of actor reputations across time

The fraction of actors who reciprocate trust is:

$$f_{\text{recip}} = p\,(x_4+x_5+x_6+x_7+x_{12}+x_{13}+x_{14}+x_{15}) + (1-p)\,(x_1+x_3+x_5+x_7+x_9+x_{11}+x_{13}+x_{15}).$$

The fraction of cheaters is:

$$f_{\text{cheat}} = 1 - f_{\text{recip}}.$$

Every actor has unknown reputation in round 1, where interactions with choosers begin. They maintain this reputation as long as they aren't trusted, with probability $1-\theta_U$ every round. Thus, in any given round $t \geq 1$, the fraction of actors with unknown reputation is:

$$\tau_U(t) = (1-\theta_U)^{t-1}.$$

Unless $\theta_U = 0$, reciprocators eventually acquire good reputation and cheaters eventually acquire bad reputation—as soon as they are trusted for the first time, exiting unknown reputation. Thus, in any given round $t \geq 1$, the fraction of actors in good and bad reputation are respectively equal to:

$$\tau_G(t) = f_{\text{recip}}(1-\tau_U),$$
$$\tau_B(t) = f_{\text{cheat}}(1-\tau_U).$$

4.3.2 Payoff of choosers who only trust given unknown reputation

Actors exit unknown reputation independently of their reciprocation strategy. This reputation does not inform choosers about likely reciprocation behavior beyond the distribution of actor strategies. In expectation, in any given round, an actor with unknown reputation is a reciprocator with probability f_{recip} and a cheater with probability f_{cheat}.

Consider a chooser who plays strategy $(1,0,0)$ (indexed by 4), trusting only actors with unknown reputation. In any given round t, he meets such an actor with probability $\tau_U(t)$, and then incurs cost γ to obtain β in return with probability f_{recip}, if the actor is a reciprocator.

His expected payoff is then:

$$U_{\text{ch},4} = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\tau_U(t)(f_{\text{recip}}\beta - \gamma).$$

Replacing $\tau_U(t)$ with its value we obtain:

$$U_{\text{ch},4} = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}(1-\theta_U)^{t-1}(f_{\text{recip}}\beta - \gamma),$$

finally yielding:

$$U_{\text{ch},4} = \frac{1-\delta}{1-\delta(1-\theta_U)}(f_{\text{recip}}\beta - \gamma). \tag{4.5}$$

4.3.3 Payoff of choosers who only trust given bad reputation

Only cheaters acquire bad reputation. A chooser who plays strategy $(0,1,0)$ (indexed by 2), trusting only actors with bad reputation, meets such actors with probability $\tau_B(t)$ in a given round t, and then incurs cost γ to never receive reciprocation.

His expected payoff is:

$$\begin{aligned}
U_{\text{ch},2} &= (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\tau_B(t)(-\gamma),\\
&= -(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}f_{\text{cheat}}(1-(1-\theta_U)^{t-1})\gamma,\\
&= -(1-\frac{1-\delta}{1-\delta(1-\theta_U)})f_{\text{cheat}}\,\gamma,\\
U_{\text{ch},2} &= -\frac{\delta\theta_U}{1-\delta(1-\theta_U)}f_{\text{cheat}}\,\gamma.
\end{aligned} \tag{4.6}$$

4.3.4 Payoff of choosers who only trust given good reputation

Conversely, a chooser who plays strategy $(0,0,1)$, trusting only actors with good reputation, is sure to receive reciprocation when he meets such actors.

His expected payoff is:

$$\begin{aligned} U_{\mathrm{ch},1} &= (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\tau_G(t)(\beta-\gamma), \\ &= (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}f_{\mathrm{recip}}(1-(1-\theta_U)^{t-1})(\beta-\gamma), \\ U_{\mathrm{ch},1} &= \frac{\delta\theta_U}{1-\delta(1-\theta_U)}f_{\mathrm{recip}}(\beta-\gamma). \end{aligned} \tag{4.7}$$

4.3.5 Payoff of any given strategy

We deduce the payoff of every chooser strategy. The expected payoff of strategy $(T_U, T_B, T_G) \in \{0,1\}^3$ is the sum of the payoff components for each reputation the chooser agrees to trust (i.e., for the bits equal to 1), which can be written as:

$$T_U\, U_{\mathrm{ch},4} \;+\; T_B\, U_{\mathrm{ch},2} \;+\; T_G\, U_{\mathrm{ch},1},$$

where $U_{\mathrm{ch},4}$, $U_{\mathrm{ch},2}$, and $U_{\mathrm{ch},1}$ are the payoffs from trusting only unknown, bad, and good reputation, respectively. Since reputations are mutually exclusive in any round, these contributions add linearly.

For example, the payoff of strategy $(1,1,0)$ is

$$U_{\mathrm{ch},6} = U_{\mathrm{ch},4} + U_{\mathrm{ch},2}.$$

5 Material for simulations of the observable investment model

We describe the strategy encodings and derive payoffs for the simulation of the observable investment model.

We run evolutionary simulations for the observable investment model using the same procedure and parameters as above.

5.1 Strategy encodings

5.1.1 Actors

Actors' decisions are unchanged, and remain independent of reputation. Their strategies are encoded the same 4-bit vectors as in the previous simulation. As before, the population is represented by the distribution of strategies X.

5.1.2 Choosers

A chooser strategy is now a 6-bit vector

$$(T_{I,U}, T_{I,B}, T_{I,G}, T_{\neg I,U}, T_{\neg I,B}, T_{\neg I,G}) \in \{0,1\}^6,$$

specifying trust decisions over the extended reputational set Ω'. For example, bit $T_{I,U}$ specifies whether to trust an actor with reputation $(\text{invested}, \text{unknown})$, while bit $T_{\neg I,G}$ specifies whether to trust an actor with reputation $(\text{didn't invest}, \text{good})$. The population is represented by the distribution of strategies Y over all $2^6 = 64$ strategies (instead of 8 previously).

5.2 Derivation of actor payoffs

5.2.1 Probabilities of being trusted

Similarly to before, for every reputation $\omega \in \Omega'$, we define the probability θ_ω that an actor is trusted when she holds that reputation. Let $\mathcal{J}_\omega$ denote the set of chooser strategy indices $j \in \{0,\ldots,63\}$ such that the ω-bit of the corresponding binary vector $T(j)$ equals 1. Then

$$\theta_\omega = \sum_{j\in\mathcal{J}_\omega} y_j.$$

For example, $\mathcal{J}_{I,U} = \{32, 33, \ldots, 63\}$ consists of all strategies with $T_{I,U} = 1$ (i.e., the most significant bit equal to 1), so

$$\theta_{I,U} = \sum_{j=32}^{63} y_j.$$

Similarly, $\mathcal{J}_{I,B} = \{j : \lfloor j/16 \rfloor \bmod 2 = 1\}$ consists of all strategies with the second bit equal to 1, and so on for the other reputations.

Using these probabilities, we calculate the payoff of all four actor action pairs.

5.2.2 Payoff of actors who don't invest and cheat

If an actor doesn't invest and cheats, she incurs no cost in round 0 and enters round 1 with reputation (didn't invest, unknown). She maintains that reputations as long as she isn't trusted; if and when she is trusted, she cheats, obtaining reputation (didn't invest, bad) from thereon.

In other words, we are in the same situations as in the previous simulation, except with different reputations.

We deduce the actor's lifetime payoff by using the same reasoning as before, replacing θ_U with $\theta_{\neg I,U}$ and θ_B with $\theta_{\neg I,B}$.

We obtain:

$$U_{\text{ac},0}(\delta) = \frac{1 - (1 - \theta_{\neg I,B})\delta}{1 - (1 - \theta_{\neg I,U})\delta}\, \delta\theta_{\neg I,U} b. \tag{5.1}$$

5.2.3 Payoff of actors with other action pairs

Similarly, replacing the reputations for the three other action pairs with the ones attained here, we obtain:

$$U_{\text{ac},1}(\delta) = \frac{1 - (1 - \theta_{\neg I,G})\delta}{1 - (1 - \theta_{\neg I,U})\delta}\, \delta\theta_{\neg I,U} (b - c_H). \tag{5.2}$$

$$U_{\text{ac},2}(\delta) = -(1-\delta)\, k + \frac{1 - (1 - \theta_{I,B})\delta}{1 - (1 - \theta_{I,U})\delta}\, \delta\theta_{I,U} b. \tag{5.3}$$

$$U_{\text{ac},3}(\delta) = -(1-\delta)\, k + \frac{1 - (1 - \theta_{I,G})\delta}{1 - (1 - \theta_{I,U})\delta}\, \delta\theta_{I,U} (b - c_L). \tag{5.4}$$

5.2.4 Payoff of any given strategy

We use the expressions above to obtain the expected payoff of any actor strategy by mixing across both possible discount factors, exactly as before.

5.3 Derivation of chooser payoffs

As in the previous simulation, and in contrast to the model, choosers are long-run players with shared discount factor δ. This allows us to obtain a single payoff metric

5.3.1 Distribution of actor reputations across time

We introduce the fractions of actors who don't invest $f_{\neg I}$, who don't invest and reciprocate trust $f_{\neg I,R}$, and don't invest and cheat $f_{\neg I,C}$, which verify

$$f_{\neg I} = f_{\neg I,R} + f_{\neg I,C},$$

and can be expressed using the actor population X. For example,

$$f_{\neg I,R} = p\,(x_4 + x_5 + x_6 + x_7) + (1-p)\,(x_1 + x_5 + x_9 + x_{13}).$$

Non-investors, of fraction $f_{\neg I}$, enter round 1 with reputation (didn't invest, unknown) and maintain it as long as they aren't trusted. In any given round $t \geq 1$, the probability of that reputation is:

$$\tau_{\neg I,U}(t) = f_{\neg I}(1 - \theta_{\neg I,U})^{t-1}.$$

Once trusted, non-investors acquire reputation (didn't invest, good) or (didn't invest, bad) depending on whether they reciprocate trust. For any $t \geq 1$, the probability of those reputations is:

$$\tau_{\neg I,G}(t) = f_{\neg I,R}(1 - \tau_{\neg I,U}),$$
$$\tau_{\neg I,B}(t) = f_{\neg I,C}(1 - \tau_{\neg I,U}).$$

We similarly introduce the fractions f_I, $f_{I,R}$, and $f_{I,C}$, and obtain, for the three remaining reputations and any given $t \geq 1$:

$$\tau_{I,U}(t) = f_I(1 - \theta_{I,U})^{t-1},$$
$$\tau_{I,G}(t) = f_{I,R}(1 - \tau_{I,U}),$$
$$\tau_{I,B}(t) = f_{I,C}(1 - \tau_{I,U}).$$

5.3.2 Chooser payoffs

A chooser who trusts only actors with reputation (invested, unknown) obtains on average:

$$U_{\text{ch},32} = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\tau_{I,U}(t)(-\gamma + \mathbb{P}(\text{reciprocate} \mid \text{invested, unknown})\,\beta).$$

Among the f_I actors who can acquire that reputation, $f_{I,R}$ of them reciprocate trust. We deduce:

$$U_{\text{ch},32} = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} f_I(1-\theta_{I,U})^{t-1}(-\gamma + \frac{f_{I,R}}{f_I}\,\beta).$$

Replacing, we obtain:

$$U_{\text{ch},32} = \frac{1-\delta}{1-\delta(1-\theta_{I,U})}(f_{I,R}\,\beta - f_I\,\gamma). \tag{5.5}$$

A chooser who trusts only actors with reputation (invested, bad) incurs cost γ each time he meets one, obtaining on average:

$$U_{\text{ch},16} = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\tau_{I,B}(t)(-\gamma).$$

Using the same steps as before, we obtain:

$$\begin{aligned} U_{\text{ch},16} &= (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} f_{I,C}(1-(1-\theta_{I,U})^{t-1})(-\gamma), \\ &= (1 - \frac{1-\delta}{1-\delta(1-\theta_{I,U})}) f_{I,C}(-\gamma), \\ &= \frac{\delta\theta_{I,U}}{1-\delta(1-\theta_{I,U})} f_{I,C}(-\gamma). \end{aligned} \tag{5.6}$$

Likewise, a chooser who trusts only actors with reputation (invested, good) obtains $\beta - \gamma$ each time he meets one, obtaining on average:

$$\begin{aligned} U_{\text{ch},8} &= (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\tau_{I,G}(t)(\beta-\gamma), \\ &= (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} f_{I,R}(1-(1-\theta_{I,U})^{t-1})(\beta-\gamma), \\ &= \frac{\delta\theta_{I,U}}{1-\delta(1-\theta_{I,U})} f_{I,R}(\beta-\gamma). \end{aligned} \tag{5.7}$$

Turning to interactions with non-investors, we simply need to replace fractions and trust probabilities for investors with the corresponding ones for non-investors. We obtain:

$$U_{\text{ch},4} = \frac{1-\delta}{1-\delta(1-\theta_{\neg I,U})}(f_{\neg I,R}\,\beta - f_{\neg I}\,\gamma), \tag{5.8}$$

$$U_{\text{ch},2} = \frac{\delta\theta_{\neg I,U}}{1-\delta(1-\theta_{\neg I,U})} f_{\neg I,C}(-\gamma), \tag{5.9}$$

$$U_{\text{ch},1} = \frac{\delta\theta_{\neg I,U}}{1-\delta(1-\theta_{\neg I,U})} f_{\neg I,R}(\beta-\gamma). \tag{5.10}$$

We deduce the payoff the other 58 strategies. The expected payoff of strategy $(T_{I,U}, T_{I,B}, T_{I,G}, T_{\neg I,U}, T_{\neg I,B}, T_{\neg I,G}) \in \{0,1\}^6$ is the sum of the payoff components for each reputation the chooser agrees to trust (i.e., for the bits equal to 1), which can be written as:

$$T_{I,U}\,U_{\text{ch},32} + T_{I,B}\,U_{\text{ch},16} + T_{I,G}\,U_{\text{ch},8} + T_{\neg I,U}\,U_{\text{ch},4} + T_{\neg I,B}\,U_{\text{ch},2} + T_{\neg I,G}\,U_{\text{ch},1}.$$

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A Demonstrations for the baseline model

A.1 Preliminary: continuation payoffs

In section 1, we define the lifetime payoff of the actor given that her discount factor is δ, $U_{\text{ac}}(\sigma \mid \delta)$. Here, we define her continuation payoffs, to aid with our proofs.

For any round $t \geq 1$, let $U^t_{\text{ac}}(a_t \mid \delta, \omega)$ be the actor's continuation payoff starting from round t as a function of her discount factor δ and her reputation ω, as well as her action in the current round $a_t \in \{\text{reciprocate}, \text{cheat}\}$. We omit σ from this notation for concision.

If the actor has a reputation that leads choosers to distrust her (e.g., if she has bad reputation and $\sigma_{\text{ch}}(\text{bad}) = \text{distrust}$), then she earns null payoff in this round, and null payoff in later rounds, because she is unable to change her reputation without being trusted (and choosers play the same pure strategy). In other words:

$$U^t_{\text{ac}}(a_t \mid \delta, \omega) \equiv 0, \qquad \forall \omega \in \sigma^{-1}_{\text{ch}}(\text{distrust}).$$

If in contrast the actor is to be trusted, her continuation payoff depends on her action a_t. If she reciprocates, she acquires good reputation, earning:

$$U^t_{\text{ac}}(\text{reciprocate} \mid \delta, \omega) \equiv (1-\delta) \times (b - c_H) + \delta \times U^{t+1}_{\text{ac}}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good}), \qquad \forall \omega \in \sigma^{-1}_{\text{ch}}(\text{trust}),$$

which notably depends on the action $\sigma_{\text{ac}}(\delta, \text{good})$ that the actor will play in the next round and whether choosers trust given good reputation. In contrast, if the actor cheats, she earns:

$$U^t_{\text{ac}}(\text{cheat} \mid \delta, \omega) \equiv (1-\delta) \times b + \delta \times U^{t+1}_{\text{ac}}(\sigma_{\text{ac}}(\delta, \text{bad}) \mid \delta, \text{bad}), \qquad \forall \omega \in \sigma^{-1}_{\text{ch}}(\text{trust}).$$

A.2 Cooperative equilibrium

We show the existence of a unique cooperative PBE.

A.2.1 Reputation incentivizes actor reciprocation

To do so, we begin by noting that choosers' strategy is uniquely determined.

Lemma A.1: Chooser strategy σ_{ch}

In a cooperative PBE, choosers trust the actor when she has unknown or good reputation, and distrust her when she has bad reputation.

Proof. Consider a pair (σ, μ), where $\sigma = (\sigma_{\text{ch}}, \sigma_{\text{ac}})$. We note first that the pair can only be cooperative if choosers trust the actor when she has an unknown reputation — otherwise, the actor is always initially distrusted, retains unknown reputation, and continues to be distrusted along the outcome path.

Second, we note that the pair can only be a cooperative PBE if choosers trust given good reputation and distrust given bad reputation — otherwise, there would be no future benefit to be gained from reciprocating rather than cheating, and since reciprocation is always costly, the actor would always cheat in a PBE. Note that an analytical proof of this is just below, by condition (a.1): on the right of both of these inequalities, the difference between the continuation payoff given that good reputation is achieved in the next round and the continuation payoff given that bad reputation is achieved in the next round can only be positive if $\sigma_{\text{ch}}(\text{good}) = \text{trust}$ and $\sigma_{\text{ch}}(\text{bad}) = \text{distrust}$. Otherwise, this difference is negative or null, and the actor strictly lose from reciprocating. □

A.2.2 The actor reciprocates depending on her discount factor

Now that σ_{ch} is determined, we turn to the actor's strategy.

Lemma A.2: Actor strategy σ_{ac}

In a cooperative PBE, whatever her current reputation, the actor reciprocates choosers' trust if and only if her discount factor δ satisfies:

$$\delta \geq \delta_0^{\text{recip.}} = \frac{c_H}{b}. \tag{1.1}$$

Proof. Consider a cooperative PBE (σ, μ). Following Lemma A.1, choosers trust given unknown and good reputation, and distrust given bad reputation. To characterize the actor's strategy, we must consider what happens given any discount factor $\delta \in \Delta$ after the actor is trusted in a given round t, including after past sequences of events that aren't supposed to happen — that is, for any reputation ω, regardless of prescribed actions.

If the actor reciprocates, she incurs cost c_H and attains good reputation. Her continuation payoff is:

$$U_{\text{ac}}^t(\text{reciprocate} \mid \delta, \omega) = (1-\delta) \times (b - c_H) + \delta \times U_{\text{ac}}^{t+1}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good}).$$

If in contrast the actor cheats, she incurs no cost but attains bad reputation, obtaining continuation payoff:

$$U_{\text{ac}}^t(\text{cheat} \mid \delta, \omega) = (1-\delta) \times b + \delta \times U_{\text{ac}}^{t+1}(\sigma_{\text{ac}}(\delta, \text{bad}) \mid \delta, \text{bad}).$$

By comparing both continuation payoffs, we deduce that in a PBE, the actor will reciprocate if and only if:

$$c_H(1-\delta) \leq \delta \times (U_{\text{ac}}^{t+1}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good}) - U_{\text{ac}}^{t+1}(\sigma_{\text{ac}}(\delta, \text{bad}) \mid \delta, \text{bad})). \tag{a.1}$$

Importantly, this condition does not depend on the actor's current reputation ω — whatever her current reputation, the actor reciprocates her partners' trust in a PBE if and only if the future value (i.e. discounted by her δ) of achieving good rather than bad reputation is larger than the immediate cost of cooperation c_H.

Here, since choosers do not trust actors with bad reputation, $U_{\text{ac}}^t(\sigma_{\text{ac}}(\delta, \text{bad}) \mid \delta, \text{bad}) = 0$. The above equation simplifies to:

$$c_H(1-\delta) \leq \delta \times U_{\text{ac}}^{t+1}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good}).$$

There are only two categories of discount factors (at most, one of them may be impossible): the set of discount factors that do not satisfy this condition, given which the actor will always cheat; and the set of discount factors that satisfy this condition, given which the actor will always reciprocate. In the second case, the actor maintains good reputation after achieving it once, and successfully engages in reciprocal cooperation with choosers in every future round. We deduce that in that case: $U^t_{\text{ac}}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good}) = U^{t+1}_{\text{ac}}(\text{reciprocates} \mid \delta, \text{good}) = (1-\delta)\sum_0^\infty \delta^t(b - c_H) = b - c_H$. Replacing, the above condition is then equivalent to:

$$(1-\delta)c_H \leq \delta \times (b - c_H).$$

Subtracting $c_H\delta$ on both sides and dividing by $b > 0$, we equivalently obtain:

$$\frac{c_H}{b} \leq \delta.$$

This proves that the actor always reciprocates if her discount factor satisfies condition (1.1). Since we have reasoned by equivalence, the set of discount factors for which the actor always cheats must be those that satisfy $\delta < \frac{c_H}{b} = \delta_0^{\text{recip.}}$ (while also being contained in the support), which proves the proposition. Note that we can also show this analytically, by replacing $U^t_{\text{ac}}(\sigma_{\text{ac}}(\delta, \text{good}) \mid \delta, \text{good})$ by $(1-\delta) \times b$ in condition (a.1), since a cheater in good reputation earns b once, and is subsequently never trusted again. □

A.2.3 Chooser posterior beliefs

We have shown that the strategy profile is uniquely determined in a cooperative PBE. We now turn to choosers' posterior beliefs, which we show can be expressed based on p and $\delta_0^{\text{recip.}}$.

Lemma A.3: Chooser posterior beliefs

In a cooperative PBE, for any reputation ω, choosers form posteriors given ω using Bayes' rule. For any $x \in \Delta$, these posteriors are given by:

$$\mu(x \mid \text{unknown}) = p(x),$$
$$\mu(x \mid \text{good}) = \frac{p(x) \times \mathbb{1}_{[\delta_0^{\text{recip.}},1)}(x)}{\mathbb{P}(\delta \geq \delta_0^{\text{recip.}})},$$
$$\mu(x \mid \text{bad}) = \frac{p(x) \times \mathbb{1}_{(0,\delta_0^{\text{recip.}})}(x)}{\mathbb{P}(\delta < \delta_0^{\text{recip.}})}.$$

Proof. Consider a cooperative PBE (σ, μ). Following Lemmas A.1-A.2, the strategy profile σ is determined, with $\delta_0^{\text{recip.}}$ separating between cases where the actor always reciprocates and cases where she always cheats.

Regardless of discount factor, the actor always begins interactions with choosers with unknown reputation in round 1, and retains unknown reputation as long as she isn't trusted (which isn't supposed to happen), which means that unknown reputation is always possible, whatever the round.

When the actor has unknown reputation, there is nothing to be learned about her discount factor. Since unknown reputation is always possible, Bayes' rule must be applied, and choosers form a posterior equal to the real distribution of discount factors, such that for any x in the support of p:

$$\mu(x \mid \text{unknown}) = p(x).$$

Choosers trust given unknown reputation. Since trust is costly and we are in a PBE, the probability that the actor reciprocates given unknown reputation must be positive: we must have $\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) > 0$, because otherwise it would be beneficial to deviate to distrusting in that case.

Good reputation is then possible, and even attained if the actor is sufficiently patient: if her discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$, along the outcome path, the actor is trusted in round 1, reciprocates, reaches good reputation in round 2, and retains it by engaging mutually beneficial cooperation with choosers thereafter.

Given good reputation, choosers can infer that the actor's discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$, and nothing more, because the actor would not attain good reputation otherwise. In other words, for any x in the support of p:

$$\mu(x \mid \text{good}) = \frac{p(x) \times \mathbb{1}_{[\delta_0^{\text{recip.}},1)}(x)}{\mathbb{P}(\delta \geq \delta_0^{\text{recip.}})}.$$

Since choosers distrust given bad reputation, we must also have $\mathbb{P}(\delta < \delta_0^{\text{recip.}}) > 0$ — otherwise, choosers would stand to gain from deviation to trusting given bad reputation, whatever their beliefs. (Recall that beliefs are defined over the support Δ of the distribution p; if every possible discount factor is above $\delta_0^{\text{recip.}}$, chooser beliefs will reflect that). Bad reputation is then possible, and attained from round 2 whenever $\delta < \delta_0^{\text{recip.}}$. Given bad reputation, choosers form the posterior given by, for any x in the support of p:

$$\mu(x \mid \text{bad}) = \frac{p(x) \times \mathbb{1}_{(0,\delta_0^{\text{recip.}})}(x)}{\mathbb{P}(\delta < \delta_0^{\text{recip.}})}.$$

□

A.2.4 Cooperative equilibrium

Now that we have determined the unique candidate cooperative PBE, we summarize our results, and derive its domain of existence. This leads us to Proposition 1.1.

Restatement of Proposition 1.1: Cooperative equilibrium

There is only one cooperative Perfect Bayesian Equilibrium. In this equilibrium, choosers trust the actor when she has an unknown or good reputation but distrust her when she has a bad reputation. Whatever her reputation, the actor reciprocates if and only if her discount factor δ satisfies:

$$\delta \geq \delta_0^{\text{recip.}} = \frac{c_H}{b}. \tag{1.1}$$

The equilibrium exists if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) < 1. \tag{1.2}$$

Proof. Consider a cooperative PBE. As we have seen, the strategy profile is determined (Lemmas A.1-A.2), and the always reciprocates if her discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$, and cheats if $\delta < \delta_0^{\text{recip.}}$.

In addition, we must have $\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) > 0$ and $\mathbb{P}(\delta < \delta_0^{\text{recip.}}) > 0$, because both reciprocation and cheating must occur with positive probability, ensuring that choosers form posteriors through Bayesian inference (Lemma A.3).

To conclude for the implication, we only need to verify that choosers do not have beneficial deviations from their strategy given their beliefs. This is immediate for good and bad reputation: since the actor is a reciprocator (resp. a cheater) with positive probability, good (resp. bad) reputation is possible, and, since actor strategy is stationary, choosers can infer with certainty that an actor whose reputation is good (resp. bad) will reciprocate (resp. cheat) again if trusted.

By trusting given good reputation, choosers obtain $u(\text{trust} \mid \text{ good }) = \beta - \alpha$ with certainty. Were they to deviate to distrusting, they would obtain 0 with certainty. This deviation isn't beneficial if and only if:

$$\gamma \leq \beta.$$

By distrusting given bad reputation, choosers obtain null payoff; were they to deviate to trusting, they would obtain $-\gamma$ with certainty, which isbn't beneficial because trust is assumed to be costly ($\gamma > 0$).

Given unknown reputation, choosers trust, and obtain on average:

$$u(\text{trust} \mid \text{unknown}) = -\gamma + \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) \times \beta,$$

since the actor will reciprocate if and only if her discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$. If choosers deviate to distrusting, they gain 0 with certainty. In other words, this deviation isn't beneficial if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}).$$

Putting everything together, we deduce that if there exists a (necessarily unique) cooperative PBE, condition (1.2) is true. (Note that this implies $\gamma < \beta$, making it beneficial to trust given good reputation.)

Conversely, we show that condition (1.2) is sufficient to guarantee that the pair (σ, μ) that we have determined in the previous lemmas constitutes a PBE. As we have seen, signalers do not have any beneficial deviations by construction. In addition, this condition guarantees the absence of beneficial deviations for choosers, as can be seen by following the same steps as before: it first guarantees the existence of discount factors for reciprocating and cheating (it implies $0 < \mathbb{P}(\delta \geq \delta_0^{\text{recip.}} < 1)$, which guarantees that discriminating based on good and bad reputation is strictly beneficial; and it also guarantees that trusting given unknown reputation is better or equivalent to distrusting. □

A.3 Other non-cooperative equilibria

As we have seen, in a cooperative equilibrium, choosers initially trust the actor given unknown reputation, in round 1. Subsequently, choosers incentivize reciprocation by trusting given good reputation, and distrusting given bad reputation.

We deduce that cooperation can fail for two reasons. First, a PBE can be non-cooperative because choosers do not incentivize reciprocation, which occurs if they adopt any other pair of actions for good and bad reputations — since reciprocation is always costly for the actor, the only way for it to occur is if choosers discriminate based on good and bad reputation. In every other case, actors always cheat, whatever the cost they face, and the only possibility in a PBE is that choosers always distrust. We obtain a trivially non-cooperative PBE, which we call the trivial equilibrium.

The other possibility is that choosers incentivize reciprocation, but are unable to initially trust, because blind trust is net costly. We obtain another non-cooperative PBE, which we call the non-cooperative equilibrium.

Below, we detail these two equilibria, which, together with the cooperative equilibrium above, comprise all the PBEs of our baseline model.

Lemma A.4: Trivial equilibrium

In the trivial equilibrium, choosers always distrust the actor, and the actor always cheats if trusted. Chooser beliefs given unknown and bad reputations are equal to the prior distribution p, and unconstrained by Bayes' rule given good reputation.
This equilibrium exists for any parameter values.

Proof. Consider the strategy profile σ whereby choosers always distrust the actor, regardless of reputation, and the actor always cheats, regardless of reputation or discount factor.

We immediately verify that there are no beneficial deviations. For choosers, since trust is costly and the actor always cheats, it is always strictly beneficial not to trust. Note that this is true whatever choosers' beliefs, since the actor consistently cheats whatever her discount factor.

Even though they have no bearing on the result, we can nevertheless derive posteriors given unknown and bad reputation, which are obtained via Bayes' rule, since both are possible (bad reputation occurs if a chooser trusts, for any δ). In those cases, choosers' posteriors are equal to the prior distribution p. They are unconstrained by Bayes' rule given good reputation, which is impossible.

For the actor, deviation to reciprocation is never beneficial. Given any δ, were the actor to be trusted in a given round, she would incur cost c_H if she reciprocated without earning any future reputational benefit, since doing so would not be rewarded with choosers' trust. □

Lemma A.5: Non-cooperative equilibrium

In the non-cooperative equilibrium, choosers trust the actor when she has good reputation, but distrust her when she has bad or unknown reputation. The actor reciprocates under the same conditions as in the cooperative equilibrium (iff $\delta \geq \delta_0^{\text{recip.}}$).
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) \leq \frac{\gamma}{\beta}. \tag{a.2}$$

Proof. Consider the proposed strategy profile. Along the outcome path, the actor is never trusted, retaining unknown reputation. Were she to be trusted, she would reciprocate if and only if her discount factor satisfies $\delta \geq \delta_0^{\text{recip.}}$.

Since choosers trust given good reputation, we deduce that the probability of reciprocation must at least be positive, because otherwise it would be beneficial to distrust in that case. We must have:

$$\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) > 0.$$

This means that good reputation is possible, and that chooser' posteriors given good reputation are derived using Bayes' rule — it is then strictly beneficial for them to trust given good reputation.

Choosers posteriors given unknown reputation are also derived using Bayes' rule, and given by p, as before. Since choosers distrust in this case, we deduce that we must have:

$$\mathbb{P}(\delta \geq \delta_0^{\text{recip.}}) \leq \frac{\gamma}{\beta}.$$

Condition (a.2) is thus necessary for the existence of this equilibrium. We show that it is sufficient, by showing that there are no beneficial deviations when it is verified. To do so, we note first that when it is verified, there are no beneficial deviations from distrusting given unknown reputation and trusting given good reputation, following the steps outlined just above.

Second, we deduce from condition (a.2) that we must have $\mathbb{P}(\delta \geq \delta_0^{\text{recip.}} < 1$, and therefore that the probability that the actor cheats if trusted is positive. As in the trivial equilibrium, bad reputation is not attained, but possible—every actor whose discount is smaller than $\delta_0^{\text{recip.}}$ attains bad reputation by playing according to σ, after a chooser deviates to trusting them given unknown reputation. Choosers' beliefs given bad reputation are thus derived using Bayes' rule, and it is strictly beneficial for them to distrust given B.

Finally, we verify that there are no beneficial deviations for the actor, by using the same steps as in the proof of Lemma A.2. ☐

B Demonstrations for the investment model

B.1 Preliminary: continuation and lifetime payoffs

We redefine the actor's continuation payoffs in any given round $t \geq 1$, taking into account the two histories for the initial round: the actor may have initially invested, leading her to consistently face low cost of reciprocation c_L, or she have initially decided not to invest.

For any round $t \geq 1$, let $U_{\text{ac}}^t(a_t \mid \delta, a_0, \omega)$ be the actor's continuation payoff starting from round t, as a function of her discount factor δ and her reputation ω, as well as her action in the current round $a_t \in \{\text{reciprocate}, \text{cheat}\}$ and the history for the initial round $h_0 \in \{\text{invested}, \text{did not invest}\}$. (Note that to determine whether a strategy profile is a SPE, we will need to consider initial histories different from the initial prescribed action).

For example, after having initially invested ($h_0 = \text{invested}$), if in round t the actor has a reputation ω that leads choosers to trust her, and if, given her discount factor δ she reciprocates given ω ($a_t = \sigma_t(\delta, \text{invested}, \omega) = \text{reciprocate}$), her continuation payoff is:

$$U_{\text{ac}}^t(\text{reciprocate} \mid \delta, \text{invested}, \omega) \equiv (1-\delta)(b - c_L) + \delta U_{\text{ac}}^t(\sigma_{\text{ac}}(\delta, \text{invested}, \text{good}) \mid \delta, \text{invested}, \text{good}),$$

for any $\omega \in \sigma_{\text{ch}}^{-1}(\text{trust})$, reflecting that the actor incurs low cost c_L to reciprocate.

We also introduce a notation for the actor's lifetime payoff, taking into account the two initial possibilities. If the actor initially invests, she pays $-k$ in the initial round, and subsequently starts interacting with choosers with unknown reputation and low cost of cooperation; her payoffs are then:

$$U_{\text{ac}}(\sigma \mid \delta) = (1-\delta)(-k) + \delta U_{\text{ac}}^t(\sigma_t(\delta, \text{invested}, \text{unknown}) \mid \delta, \text{invested}, \text{unknown}), \quad \forall \delta \in \sigma_0^{-1}(\text{invest}).$$

If in contrast the actor does not initially invest, we have:

$$U_{\text{ac}}(\sigma \mid \delta) = 0 + \delta U_{\text{ac}}^t(\sigma_t(\delta, \text{did not invest}, \text{unknown}) \mid \delta, \text{did not invest}, \text{unknown}),$$

for any $\delta \in \sigma_0^{-1}(\text{do not invest})$. For concision, we denote the actor's payoff $U_{\text{ac}}^0(\text{invest} \mid \delta)$ in the first case, and $U_{\text{ac}}^0(\text{do not invest} \mid \delta)$ in the second.

B.2 Cooperative equilibrium

We show the existence of a unique cooperative PBE.

B.2.1 Cooperation occurs as in the baseline equilibrium

To do so, we begin by noting that behavior in trust games in analogous to that obtained in the baseline equilibrium: choosers trust the actor when he has unknown or good reputation but distrust her when has bad reputation, and the actor reciprocates based on her discount factor δ and the cost-to-benefit ratio of reciprocation, which is either $\frac{c_H}{b}$ or $\frac{c_L}{b}$.

Lemma B.1: Chooser strategy σ_{ch}

In a cooperative PBE, choosers trust the actor when she has unknown or good reputation, and distrust her when she has bad reputation.

Lemma B.2: Actor trust game strategy σ_t

In a cooperative PBE, whatever her current reputation, the actor reciprocates choosers' trust if and only if her discount factor δ is greater than or equal to the cost-to-benefit ratio of cooperation, which is equal to $\frac{c_L}{b}$ if she initially invested, and $\frac{c_H}{b}$ if she did not. The actor always cheats if she is impatient ($\delta < \frac{c_L}{b}$), always reciprocates if patient ($\delta \geq \frac{c_H}{b}$), and if her discount factor is intermediate ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$), she reciprocates if she initially invested but cheats if she initially opted out of investing.

Proof. The proof of Lemma B.1 is immediate: as in the baseline model, there are only three reputational states, and a cooperative PBE requires that choosers play according to this strategy, as deduced using the same steps than in the proof for Lemma A.1).

The proof of Lemma B.2 is also immediate, and uses the continuation payoffs introduced at the beginning of this section and the steps of the proof for Lemma A.2. There are two cases: after having opted out of investing ($h_0 =$ did not invest), the actor consistently faces high cost of reciprocation c_H; we obtain the same continuation payoffs as in the baseline model, leading us to conclude that the actor will reciprocate if and only if her discount factor satisfies $\delta \geq \frac{c_H}{b}$ in a PBE.

In contrast, after having invested ($h_0 =$ invested), the actor consistently low cost of reciprocation c_L; replacing c_H with c_L in condition (a.1), we deduce that she will reciprocate if and only if:

$$c_L(1-\delta) \leq \delta(U^t_{\text{ac}}(\sigma_t(\delta, \text{invested}, \text{good}) \mid \delta, \text{invested}, \text{good}) - U^t_{\text{ac}}(\sigma_t(\delta, \text{invested}, \text{bad}) \mid \delta, \text{invested}, \text{bad})).$$

Using the same steps than in the proof for Lemma A.2, we deduce that the actor will reciprocate in this case if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$. □

B.2.2 Actor investment

Now that σ_{ch} and σ_t are determined, we turn to actor's initial investment strategy σ_0. Following the previous analysis, there are (at most) three possible cases for actor behavior in trust games. Each case corresponds to one of the following lemma.

Lemma B.3: Actor investment strategy σ_0 given $\delta < \frac{c_L}{b}$

In a cooperative PBE, if the actor is impatient ($\delta < \frac{c_L}{b}$) she does not invest in reducing the cost of cooperation.

Lemma B.4: Actor investment strategy σ_0 given $\delta \geq \frac{c_H}{b}$

In a cooperative PBE, if the actor is patient ($\delta \geq \frac{c_H}{b}$), she invests in reducing the cost of cooperation if and only if her discount factor also verifies:

$$\delta \geq \delta_1^{\text{invest,high}} \equiv \frac{k}{k + c_H - c_L}. \tag{b.1}$$

In particular, if the actor is patient she is certain to initially invest in reducing the cost of cooperation if:

$$\frac{c_H}{b} \geq \delta_1^{\text{invest,high}}. \tag{b.2}$$

Lemma B.5: Actor investment strategy σ_0 given $\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$

In a cooperative PBE, if the actor has intermediate patience ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$), she invest in reducing the cost of cooperation if and only if her discount factor satisfies:

$$\delta \geq \delta_1^{\text{invest,medium}} \equiv \frac{c_L - k + \sqrt{(k - c_L)^2 + 4kb}}{2b}. \tag{b.3}$$

In particular, if the actor has intermediate patience than she is certain not to initially invest in reducing the cost of cooperation if:

$$\frac{c_H}{b} \leq \delta_1^{\text{invest,high}}. \tag{b.2'}$$

Proof. Consider a cooperative PBE (σ, μ). Following Lemma B.2, there are three possible cases. First case: the actor is impatient, i.e., her discount factor satisfies $\delta < \frac{c_L}{b}$ (assuming such a discount factor is possible; otherwise, Lemma B.3 is moot).

In this case, the actor will cheat when trusted in all rounds $t \geq 1$, whatever her initial decision. Along the outcome path, she obtains b in round 1 (the first chooser trusts her, when her reputation is empty), and null payoff in every round $t \geq 2$ (she has bad reputation thereafter). Since investing is costly and does not affect the payoff of a cheator, the actor strictly benefits from opting out of investing in round 0. In a PBE, impatient actors do not invest in reducing the cost of cooperation; this proves the first lemma.

Second case: the actor is patient ($\delta \geq \frac{c_H}{b}$). In this case, the actor will reciprocate in all rounds $t \geq 1$, whatever her initial decision. As a result, she is trusted by every chooser along the outcome path. If she invests in round 0, she obtains payoff:

$$U_{\text{ac}}^0(\text{invest} \mid \delta) = (1 - \delta) \times (-k) + \delta \times (b - c_L), \qquad \forall \delta \geq \frac{c_H}{b},$$

and if she does not invest in round 0, she obtains:

$$U_{\text{ac}}^0(\text{do not invest} \mid \delta) = 0 + \delta \times (b - c_H), \qquad \forall \delta \geq \frac{c_H}{b}.$$

By comparing between both payoffs, we deduce that in a PBE, the actor invests in reducing the cost of cooperation if and only if the immediate cost of doing so is smaller than the long-term benefit of cooperating with a low rather than a high cost of reciprocation; that is, iff:

$$(1 - \delta)k \leq \delta((b - c_L) - (b - c_H)).$$

Simplifying, the above inequality is equivalent to:

$$(1 - \delta)k \leq \delta(c_H - c_L),$$

which, re-arranging, is also equivalent to:

$$\frac{k}{k + c_H - c_L} \leq \delta.$$

We recognize $\delta_1^{\text{invest,high}}$ on the left, proving condition (b.1), and the upper part of the second lemma. In particular, if the actor is patient she is certain to initially invest if the above inequality is verified for $\delta = \frac{c_H}{b}$ (the smallest possible discount factor for patience), that is if:

$$\delta_1^{\text{invest,high}} \leq \frac{c_H}{b}. \tag{b.2}$$

This proves the remaining part of the second lemma. Finally, we turn to the case of a actor of intermediate patience ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$). This actor reciprocates choosers' trust in all round $t \geq 1$ if she initially invests, gaining:

$$U_{\text{ac}}^0(\text{invest} \mid \delta) = (1 - \delta) \times (-k) + \delta \times (b - c_L), \qquad \forall \delta : \frac{c_L}{b} \leq \delta < \frac{c_H}{b}.$$

If in contrast she does not invest in round 0, she cheats once and is never trusted again, gaining:

$$U^0_{\text{ac}}(\text{do not invest} \mid \delta) = 0 + \delta \times (1-\delta)b, \qquad \forall \delta : \frac{c_L}{b} \leq \delta < \frac{c_H}{b}.$$

Comparing between both payoffs, we deduce that in a PBE, the actor will initially invest in this case if and only if:

$$\delta(1-\delta)b \leq -k(1-\delta) + \delta(b - c_L).$$

Re-arranging we equivalently obtain:

$$0 \leq \delta^2 b + \delta(k - c_L) - k.$$

The above right is a second-degree polynomial in δ, which we denote $Q(\delta)$. Over the considered interval, its derivative is positive: $Q'(\delta) = 2\delta b + k - c_L \geq 2c_L + k - c_L > 0$.

We note that Q is negative for $\delta = \frac{c_L}{b}$:

$$Q(\frac{c_L}{b}) = (\frac{c_L}{b})^2 b + (\frac{c_L}{b})(k - c_L) - k = (\frac{c_L}{b} - 1)k < 0,$$

and that:

$$Q(\frac{c_H}{b}) = \frac{c_H}{b}(c_H + k - c_L) - k = [\frac{c_H}{b} - \frac{k}{k + c_H - c_L}] \times [k + c_H - c_L].$$

Since $k + c_H - c_L$ is always positive, $Q(\frac{c_H}{b})$ is of the same sign as $\frac{c_H}{b} - \frac{k}{k+c_H-c_L}$, the difference between the high cost-to-benefit ratio $\frac{c_H}{b}$ and the previously defined $\delta_1^{\text{invest,high}}$. The expression $Q(\frac{c_H}{b})$ is thus positive if and only if:

$$\delta_1^{\text{invest,high}} < \frac{c_H}{b}.$$

Following the intermediate value theorem, there are two possibilities for the actor when she has intermediate patience. If the above condition does not hold, that is, if:

$$\delta_1^{\text{invest,high}} \geq \frac{c_H}{b}, \tag{b.2'}$$

then Q is strictly negative over the entire interval $[\frac{c_L}{b}, \frac{c_H}{b})$ — in this case, this actor always initially decides not to invest. This proves the bottom part of the third lemma.

If in contrast (b.2') is false, that is if the original strict inequality is true, then $Q(\frac{c_H}{b}) > 0$, and Q admits a unique root belonging to the interval $(\frac{c_L}{b}, \frac{c_H}{b})$. The discriminant of Q is equal to: $(k - c_L)^2 + 4kb$. The root is given by:

$$\delta_1^{\text{invest,medium}} = \frac{c_L - k + \sqrt{(k - c_L)^2 + 4kb}}{2b},$$

as defined in condition (b.3). The expression $Q(\delta)$ is then negative when $\frac{c_L}{b} \leq \delta < \delta_1^{\text{invest,medium}}$, and positive or null when $\delta_1^{\text{invest,medium}} \leq \delta < \frac{c_H}{b}$. This proves the upper part of the third, and final, lemma. □

B.2.3 Graph representing actor strategy

In the previous two subsections, we have determined the actor's strategy $\sigma_{\text{ac}} = (\sigma_0, \sigma_t)$, and shown that her investment and trust game behavior depend only on her discount factor δ, as compared to the cost-to-benefit ratios of cooperation, $\frac{c_L}{b}$ and $\frac{c_H}{b}$, and two critical values, that we denoted $\delta_1^{\text{invest,medium}}$ and $\delta_1^{\text{invest,high}}$.

Indifference between investment and opting out occurs if the actor is patient or of intermediate patience, depending on whether condition (b.2) is satisfied; that is, depending on whether we have:

$$\frac{c_H}{b} \geq \frac{k}{k + c_H - c_L},$$

which, re-arranging, is equivalent to:

$$bk < c_H(k + c_H - c_L).$$

Re-arranging once more, this condition is equivalent to:

$$(b - c_H)k < c_H(c_H - c_L),$$

which, assuming $c_H < b$, is equivalent to:

$$k < \frac{c_H(c_H - c_L)}{b - c_H}.$$

We use this condition to plot Supplementary Figure 2, in which we represent the actor's behavior, obtaining two cases: to the left, when k is relatively small, $\delta_1^{\text{invest,medium}}$ separates between discount factors for which the actor invests and reciprocates, and those for which the actor opts out and cheats. To the right, when k is relatively large, we obtain three possible behaviors for the actor along the outcome path.

Note that this latter case is unlikely in our case of interest, where c_H is large, hindering cooperation in the baseline model and justifying the introduction of this extension. Were for instance we to have $c_H \geq b$, the upper line of Supplementary Figure 2 could be ignored, and we would only see the case currently represented on the left.

This is also true with our default parameters, used for Figures 2-3 (main text) and Supplementary Figures 2-3. With these parameters, we obtain $\frac{c_H(c_H - c_L)}{b - c_H} = 3.6$, meaning that we would have to extend the below figure much further to the right. Here, to visually represent all cases for the actor's strategy, we adopt different parameters, which are more conducive to cooperation even in the absence of investment.

B.2.4 Chooser posterior beliefs

We have shown that the strategy profile is uniquely determined in a cooperative PBE. In particular, we have shown that the actor initially invests if and only if her discount factor δ is greater than or equal to:

$$\delta_1^{\text{invest}} = \max\{\delta_1^{\text{invest, medium}}, \delta_1^{\text{invest, high}}\}, \tag{2.1}$$

and reciprocates when trusted if and only if her discount factor δ is greater than:

$$\delta_1^{\text{recip.}} = \min\{\delta_1^{\text{invest,medium}}, \frac{c_H}{b}\}, \tag{2.2}$$

as reiterated by the below lemma. We now turn to choosers' posterior beliefs, which we show can be expressed based on p and $\delta_1^{\text{recip.}}$.

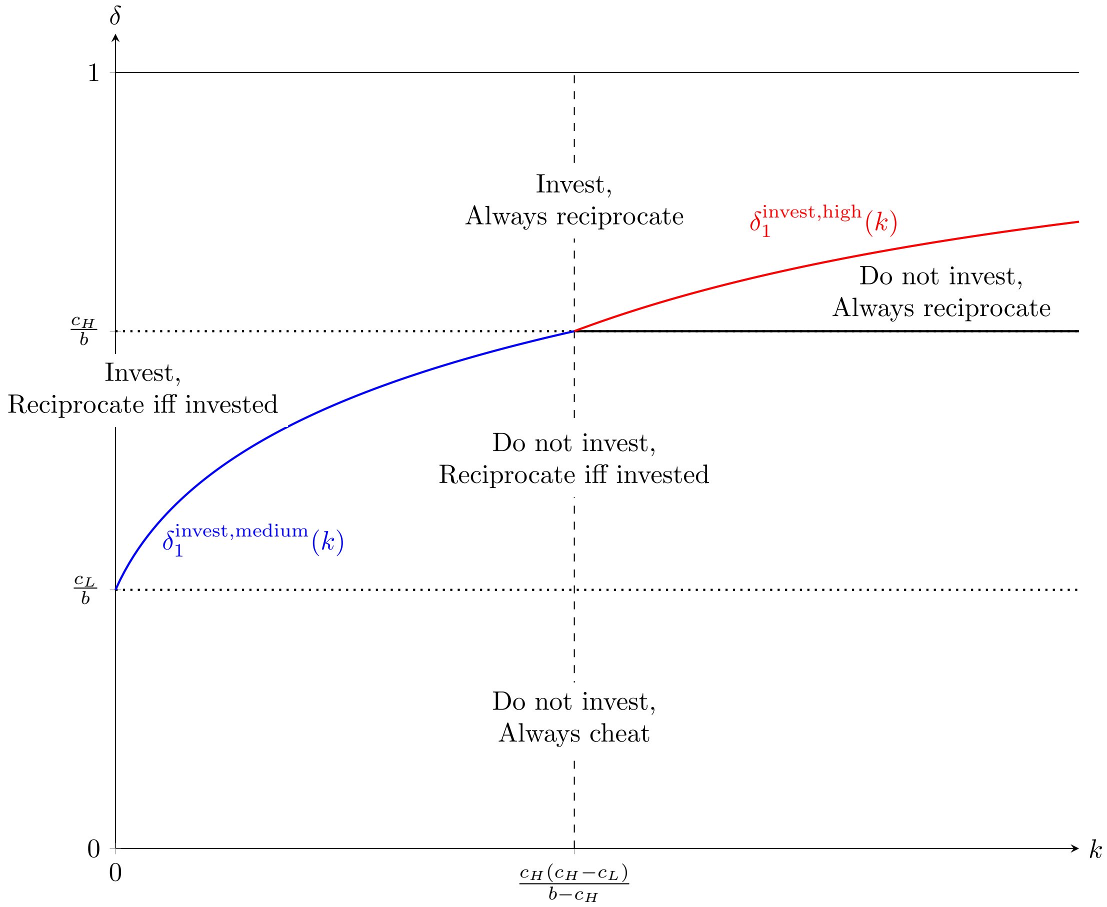
Lemma B.6: Actor cooperative behavior

In a cooperative PBE, the threshold $\delta_1^{\text{recip.}}$ separates between reciprocation and cheating. Along the outcome path, the actor reciprocates choosers' trust if and only if her discount factor satisfies $\delta \geq \delta_1^{\text{recip.}}$.

Proof. Immediate from Lemmas B.3-B.5. As we have seen there are two cases. In the first case, if $\frac{c_H}{b} > \delta_1^{\text{invest,high}}$ (to the left of Figure 2), the actor reciprocates along the outcome path if and only if she initially invests, and the threshold $\delta_1^{\text{invest,medium}}$ separates between investment and opting out, and therefore between reciprocation and cheating.

In the second case, if $\frac{c_H}{b} \leq \delta_1^{\text{invest,high}}$ (to the right of Figure 2), the actor reciprocates along the outcome path if and only if she can afford even the high cost. Investing is then prohibitively costly given intermediate patience ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$). In this case, the baseline cost-to-benefit ratio $\frac{c_H}{b}$ separates between reciprocation and cheating, the threshold for investment being even higher.

Since $\frac{c_H}{b} > \delta_1^{\text{invest,high}} \iff \frac{c_H}{b} > \delta_1^{\text{invest,medium}}$ (see demonstration of Lemmas B.3-B.5), we deduce that $\delta_1^{\text{recip.}}$ separates between reciprocation and cheating more generally, in both cases. □



Supplementary Figure 2: Visual representation of the actor's strategy as a function of the cost of investing k (x-axis) and her discount factor δ (y-axis). For any given investment cost k, an actor with discount factor behaves according to the region she falls in. The dashed vertical line $k = \frac{c_H(c_H-c_L)}{b-c_H}$, which is equivalent to $\delta_1^{\text{invest,high}} = \delta_0^{\text{recip.}} = \frac{c_H}{b}$, separates between two cases for actor behavior along the outcome path. **First case**: when $k < \frac{c_H(c_H-c_L)}{b-c_H}$, the actor does not invest and subsequently cheats along the outcome path if $\delta < \delta_1^{\text{invest,medium}}$; if in contrast $\delta \geq \delta_1^{\text{invest,medium}}$, the actor invests and then reciprocates. The blue line representing $\delta_1^{\text{invest,medium}}(k)$ thus separates between two types of behavior — in contrast, the dotted vertical lines at $\frac{c_L}{b}$ and $\frac{c_H}{b}$ separate between different strategies which yield the same behavior along the outcome path. **Second case**: when $k \geq \frac{c_H(c_H-c_L)}{b-c_H}$, three types of behavior become possible. Under the black line ($\delta < \frac{c_H}{b}$), the actor does not invest and subsequently cheats along the outcome path, above the red line ($\delta \geq \delta_1^{\text{invest,high}}$), she invests and subsequently reciprocates; and in between the two ($\frac{c_H}{b} \leq \delta < \delta_1^{\text{invest,high}}$), she does not invest but nevertheless reciprocates. We take $b = 1$, $c_L = 1/3$, $c_H = 2/3$, and vary k between 0 and 1.4, and δ between 0 and 1. With these parameters, the dashed vertical line is at $k = 2/3$.

Lemma B.7: Chooser posterior beliefs

In a cooperative PBE, for any reputation ω, choosers form posteriors given ω using Bayes' rule. For any x in the support of p, these posteriors are given by:

$$\mu(x \mid \text{unknown}) = p(x),$$

$$\mu(x \mid \text{good}) = \frac{p(x) \times \mathbb{1}_{[\delta_1^{\text{recip.}},1)}(x)}{\mathbb{P}(\delta \geq \delta_1^{\text{recip.}})},$$

$$\mu(x \mid \text{bad}) = \frac{p(x) \times \mathbb{1}_{(0,\delta_1^{\text{recip.}})}(x)}{\mathbb{P}(\delta < \delta_1^{\text{recip.}})}.$$

Proof. Consider a cooperative PBE (σ, μ). Following the previous lemmas, σ is determined, with $\delta_1^{\text{recip.}}$ separating between cases where the actor reciprocates along the outcome path, and cases where she cheats along the outcome path.

Based on this result (Lemma B.6), we can use the same steps as in Lemma A.3 to derive all three posteriors using Bayes' rule. In short, unknown reputation is always possible, and since choosers trust given unknown reputation, the likelihood of reciprocation along the outcome path must be positive, leading to $\mathbb{P}(\delta \geq \delta_1^{\text{recip.}}) > 0$.

We deduce that good reputation is possible, and even attained by the actor when she is sufficiently patient, allowing us to deduce choosers' posterior in that case as well. Finally, since choosers distrust given bad reputation, we deduce that cheating must also occur along the outcome path with positive probability: we deduce $\mathbb{P}(\delta < \delta_1^{\text{recip.}}) > 0$ and derive choosers' last posterior. ☐

B.2.5 Cooperative equilibrium

Now that we have determined the unique candidate cooperative PBE, we summarize our results, and derive its domain of existence.

Restatement of Proposition 2.1: Cooperative equilibrium

There is only one cooperative Perfect Bayesian Equilibrium. In this equilibrium, choosers trust the actor when she has an unknown or good reputation but distrust her when she has a bad reputation. The actor initially invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_1^{\text{invest}} = \max\{\frac{c_L - k + \sqrt{(k-c_L)^2 + 4kb}}{2b}, \frac{k}{k + c_H - c_L}\}. \tag{2.1}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$; after not investing, she reciprocates if and only if $\delta \geq \frac{c_H}{b}$. Along the outcome path, the actor reciprocates if and only if:

$$\delta \geq \delta_1^{\text{recip.}} = \min\{\frac{c_L - k + \sqrt{(k-c_L)^2 + 4kb}}{2b}, \frac{c_H}{b}\}. \tag{2.2}$$

This equilibrium exists if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_1^{\text{recip.}}) < 1. \tag{2.3}$$

Proof. Consider a cooperative PBE. As we have seen, the actor's strategy is determined as a function of her discount factor and the thresholds introduced in Lemmas B.2-B.6. In particular, the threshold $\delta_1^{\text{recip.}}$ separates between reciprocator and cheater types, which must both exist with positive probability (as we saw in the proof of Lemma B.7). In other words, we must have:

$$0 < \mathbb{P}(\delta \geq \delta_1^{\text{recip.}}) < 1.$$

Choosers' posteriors and strategy are uniquely determined (Lemma B.1 and Lemma B.7). To conclude for the implication, we only need to verify that choosers do not have beneficial deviations from their strategy given their beliefs. This is immediate for good and bad reputation: since reciprocating and cheating types are drawn with positive chance, good and bad reputation are both possible, and even attained, because choosers trust given unknown reputation; and, since actor strategy is stationary, choosers can infer with certainty that an actor whose reputation is good (resp. bad) will reciprocate (resp. cheat) again if trusted.

As in the proof of Proposition 1.1, we deduce that there are no beneficial deviations for choosers given good reputation if and only if:

$$\gamma \leq \beta,$$

and that distrusting given bad reputation is always beneficial (because $\gamma > 0$).

Given unknown reputation, choosers incur cost γ and receive benefit β with probability $\mathbb{P}(\delta \geq \delta_1^{\text{recip.}}$. Deviation to distrusting is then non-beneficial if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_1^{\text{recip}}).$$

Putting everything together, we see that if there exists a cooperative PBE, then it is unique, and condition (2.3) must hold.

Conversely, we verify that this condition is sufficient to guarantee that the pair (σ, μ) that we have determined in the previous lemmas constitutes a PBE: by constructions, there are no beneficial deviations for the actor, whatever her discount factor, and retracing our steps above, there are no beneficial deviations for choosers. □

B.3 Other non-cooperative equilibria

As in the baseline model, we obtain two non-cooperative equilibria, which we once again call the trivial equilibrium and the non-cooperative equilibrium. In both, the actor never initially invests, since she can expected never to be trusted. These equilibria are analogous to the ones for the baseline model, and obtained under the same conditions.

Lemma B.8: Trivial equilibrium

In the trivial equilibrium, choosers always distrust the actor, the actor never invests and always cheats if trusted. Chooser beliefs given unknown and bad reputations are equal to the prior distribution p, and unconstrained by Bayes' rule given good reputation.
This equilibrium exists for any parameter values.

Lemma B.9: Non-cooperative equilibrium

In the non-cooperative equilibrium, choosers trust the actor when she has good reputation, but distrust her when she has bad or unknown reputation.
The actor never invest, and would reciprocate if trusted under the same conditions as in the baseline equilibrium, if and only if her discount factor δ is greater than or equal to the cost-to-benefit ratio of cooperation, which is equal to $\frac{c_L}{b}$ if she initially invested, and $\frac{c_H}{b}$ if she did not.
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \frac{c_H}{b}) \leq \frac{\gamma}{\beta}. \tag{a.2}$$

Proof. Immediate for both equilibria, using the reasoning from Appendix A and Lemma B.2, and the fact that investment cannot occur in a PBE, since it is immediately costly and cannot improve the actor's future payoff, because she is never trusted along the outcome path. □

C Demonstrations for the observable investment model

We show the existence of two types of cooperative PBEs, defined according to choosers' actions: signaling equilibria, where choosers discriminate based on investment behavior; and the universal trust equilibrium, where choosers trust regardless of initial investment.

C.1 Signaling equilibria

We begin by studying the signaling equilibrium of this extension, in which choosers trust the actor when her reputation is $(\text{invested}, \text{unknown})$ and distrust her given $(\text{didn't invest}, \text{unknown})$.

C.1.1 Cooperation

To do so, we begin by noting that choosers must discriminate according to trust game reputation after the actor invests, without which there would be no cooperation with the actor along the outcome path.

There is no such restriction if the actor does not invest, since this leads her never to be trusted along the outcome path. As a result, there are two possibilities, which are consistent with the logic of the two non-cooperative equilibria in the previous models: either choosers always distrust if the actor doesn't initially invest, regardless of hypothetical future trust game behavior, in which case the actor always cheats; or choosers trust given (didn't invest, good) but distrust given (didn't invest, bad), and, after not investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_H}{b}$.

Lemma C.1: Chooser strategy $\sigma_{\mathbf{ch}}$

In a signaling PBE, choosers trust the actor if her reputation is (invested, unknown) or (invested, good) but distrust her if her reputation is (didn't invest, unknown), (invested, bad) or (invested, good).
There are two possibilities for the remaining reputation: choosers may trust or distrust given (didn't invest, good), leading to two different signaling equilibria.

Lemma C.2: Actor trust game strategy σ_t

In a signaling PBE, whatever her current reputation, the actor reciprocates choosers' trust after initially investing if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$.
There are two possibilities given that the actor did not initially invest. If choosers distrust given (didn't invest, good), the actor always cheats; if choosers trust given (didn't invest, good), the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_H}{b}$.

Proof. By definition, in a signaling equilibrium, choosers trust if the actor invested and distrust if she did not. In addition, and analogously to the proof for Lemma A.1, choosers must trust given (invested, good) and distrust (invested, bad): since the equilibrium is cooperative, there exists a set of possible discount factors for which the actor initially invests, is trusted, and subsequently reciprocates, which means that choosers must discriminate according to trust game behavior in this case — otherwise, there would be no incentive to ever incur the low cost of reciprocation $c_L > 0$ for the actor.

In the subgame occurring after the actor doesn't invest, there are at most two possibilities, as we have seen: either choosers discriminate according to actor reputation, incentivizing her to reciprocate, which she does iff $\delta \geq \frac{c_H}{b}$; or they don't, in which case, the only possibility is that choosers always distrust and that the actor always cheats. In both cases, neither player can benefit from deviation, meaning that they can both occur in a SPE.

Finally, as seen in Lemma B.2, the actor reciprocates after investing if and only if her discount factor satisfites $\delta \geq \frac{c_L}{b}$. □

C.1.2 Actor investment

We now turn to actor's initial investment strategy σ_0. Following the previous analysis, there are two cases at most: $\delta < \frac{c_L}{b}$, in which case the actor can be expected to cheat even after investing; and $\delta \geq \frac{c_L}{b}$, in which case she can be expected to reciprocate after investing. In both cases, the actor earns null payoff if she initially opts out of investing, since this leads her never to be trusted along the outcome path.

Lemma C.3: Actor investment strategy σ_0 given $\delta < \frac{c_L}{b}$

In a signaling PBE, if the actor's discount factor satisfies $\delta < \frac{c_L}{b}$, she does not initially invest if and only if her discount factor also satisfies:

$$\delta \geq \frac{k}{b}. \tag{c.1}$$

In particular, after investing, the actor reciprocates with certainty if:

$$k \geq c_L. \tag{c.2}$$

Lemma C.4: Actor investment strategy σ_0 given $\delta \geq \frac{c_L}{b}$

In a signaling PBE, if the actor's discount factor satisfies $\delta \geq \frac{c_L}{b}$, she initially invests if and only if her discount factor also satisfies:

$$\delta \geq \frac{k}{k+b-c_L}. \tag{c.3}$$

In particular, every reciprocating type initially invests if:

$$k \leq c_L. \tag{c.2'}$$

Proof. Consider a signaling PBE (σ, μ). By definition, choosers discriminate based on the initial investment. Any actor who opts not to invest is never trusted, and gains null payoff:

$$U_{\text{ac}}^0(\text{don't invest} \mid \delta) = 0, \qquad \forall \delta.$$

Following Lemma C.2, there are two possible cases after investing. In the first case, the actor's discount factor satisfies $\delta < \frac{c_L}{b}$, meaning that she will cheat in all future rounds $t \geq 1$ even if she invests in round 0. If she invests, she obtains:

$$U_{\text{ac}}^0(\text{invest} \mid \delta) = (1-\delta)(-k+\delta b), \qquad \forall \delta < \frac{c_L}{b}.$$

Comparing both payoffs, we deduce that in a PBE, the actor initially invests when her discount factor is smaller than $\frac{c_L}{b}$ if and only if:

$$\delta \geq \frac{k}{b}. \tag{c.1}$$

In particular, there are no actor types that invest and then cheat if the above condition is false for every $\delta < \frac{c_l}{b}$; replacing δ with $\frac{c_L}{b}$, we deduce condition (c.2).

The second case, the actor's discount factor satisfies $\delta \geq \frac{c_L}{b}$, meaning that she will reciprocate in all future rounds $t \geq 1$ after investing in round 0. If this actor invests, she obtains:

$$U_{\text{ac}}^0(\text{invest}) \mid \delta) = (1-\delta) \times (-k) + \delta \times (b - c_L), \qquad \forall \delta \geq \frac{c_L}{b}.$$

Comparing this payoff with 0, the payoff of not investing, we deduce that in a PBE, the actor initially invests when her discount factor is great than or equal to $\frac{c_L}{b}$ if and only if:

$$\delta(b - c_L) \geq (1-\delta)k,$$

which, re-arranging, is equivalent to:

$$\delta \geq \frac{k}{k+b-c_L} \tag{c.3}$$

In particular, there are no actor types that don't invest but would reciprocate if they had if the above condition is true for every $\delta \geq \frac{c_L}{b}$; replacing δ with $\frac{c_L}{b}$, we deduce that this occurs if:

$$\frac{c_L}{b}(b - c_L) \geq (1 - \frac{c_L}{b})k.$$

Dividing by $1 - \frac{c_L}{b}$ on both sides, we deduce that every reciprocator type initially invests if:

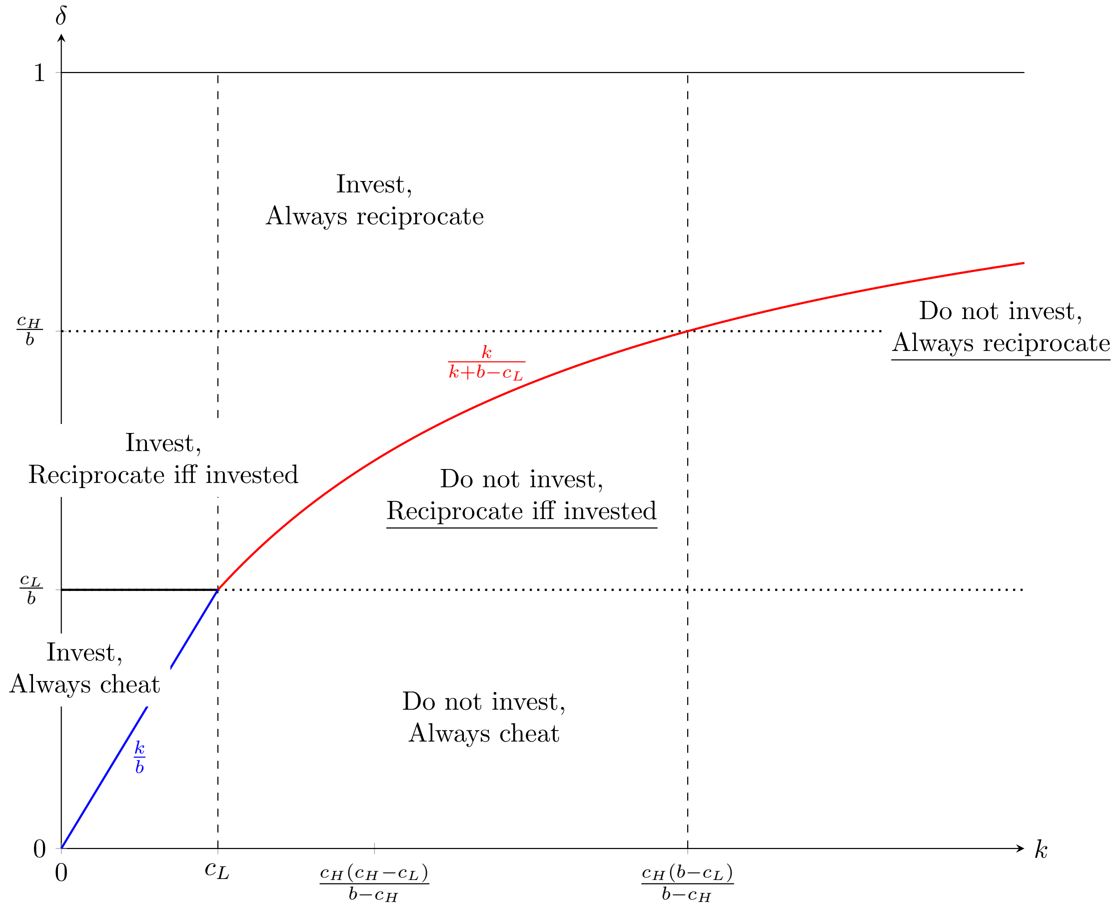
$$c_L \geq k. \tag{c.2'}$$

□

C.1.3 Graph representing actor strategy

In the previous two sections, we have determined the actor's strategy, finding a unique strategy for investment and reciprocation after investing, and two possibilities for reciprocation after not investing. Since not investing leads to being distrusted, the actor's behavior along the outcome path is the same in both cases.

We represent the actor's strategy in Supplementary Figure 3, using the same parameter values and space that we used in Supplementary Figure 2.



Supplementary Figure 3: Visual representation of the actor's strategy as a function of the cost of investing k (x-axis) and her discount factor δ (y-axis), in the second signaling equilibrium (the actor's strategy in the first is obtained by replacing both underlined terms by 'Always cheat'). For any given investment cost k, an actor with discount factor behaves according to the region she falls in. The dashed vertical line $k = c_L$ separates between two cases for actor behavior along the outcome path. **First case**: when $k < c_L$, the signal can be dishonest. The actor does not invest and would cheat if trusted given $\delta < \frac{k}{b}$, invests and cheats given $\frac{k}{b} \leq \delta < \frac{c_L}{b}$, and invests and reciprocates given $\delta \geq \frac{c_L}{b}$. **Second case**: when $k \geq c_L$, the signal is automatically honest. The actor does not invest and is never trusted given $\delta < \frac{k}{k+b-c_L}$, and invests and reciprocates given $\delta \geq \frac{k}{k+b-c_L}$. Note that there can be non-investing reciprocating types in this case, so long as we also have $k > \frac{c_H(b-c_L)}{b-c_H}$. We take $b = 1$, $c_L = 1/3$, $c_H = 2/3$, as in Figure 2. We vary k between 0 and 2.05 however, rather than 0 and 1.4, to visualize non-investing reciprocating types. The value $\frac{c_H(c_H-c_L)}{b-c_H} = 2/3$, which separates between the two types of behavior in extension 1, is indicated on the x-axis for comparison. With these values, the first vertical line is at $k = 1/3$, and the second at $k = 4/3$.

C.1.4 Signaling equilibrium

Now that we have determined the existence of two signaling PBEs, we summarize our results, and derive their domains of existence, in propositions 3.2 and 3.3. Proposition 3.1 applies to both signaling equilibria.

Restatement of Proposition 3.1: Signaling equilibria

In any signaling equilibrium, choosers trust the actor when her reputation is (invested, unknown) or (invested, good), but distrust her when her reputation is (invested, bad), (didn't invest, unknown), or (didn't invest, bad). The actors initially invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_{2,s}^{\text{invest}} \equiv \min\{\frac{k}{b}, \frac{k}{k+b-c_L}\}. \tag{3.1}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$, whatever her reputation. Her behavior after not investing depends on the specific equilibrium. Along the outcome path, the actor reciprocates if and only if:

$$\delta \geq \delta_{2,s}^{\text{recip.}} \equiv \max\{\frac{c_L}{b}, \frac{k}{k+b-c_L}\}. \tag{3.2}$$

Restatement of Proposition 3.2: First signaling equilibrium

In the first signaling equilibrium, choosers distrust the actor when her reputation is (didn't invest, good). After not investing, the actor always cheats.
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}}), \tag{3.3}$$

$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.4}$$

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest.}}). \tag{3.5}$$

Restatement of Proposition 3.3: Second signaling equilibrium

In the second signaling equilibrium, choosers trust the actor when her reputation is (didn't invest, good). After not investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_H}{b}$.
This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}}), \tag{3.3}$$

$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.4}$$

$$0 < \mathbb{P}(\delta \geq \frac{c_H}{b}), \tag{3.6}$$

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}). \tag{3.5}$$

$$\delta_{2,s}^{\text{invest}} \leq \frac{c_H}{b} \text{ or } \mathbb{P}(\delta \geq \frac{c_H}{b} \mid \delta < \delta_{2,s}^{\text{invest}}) < \frac{\gamma}{\beta}. \tag{3.7}$$

Proof. As we have seen, there are two strategy profiles of interest. In both cases, choosers trust given (invested, unknown) or (invested, good), and distrust given (didn't invest, unknown), (invested, bad) or (didn't invest, bad). In addition, the actor invests if and only if her discount factor satisfies $\delta \geq \delta_{2,s}^{\text{invest}}$, and reciprocates along the outcome path if and only if $\delta \geq \delta_{2,s}^{\text{recip.}}$, as can be seen in Supplementary Figure 3 — Proposition 3.1 is a direct consequence of the previously proven lemmas.

We begin with the first signaling equilibrium, in which choosers distrust given (didn't invest, good) and the actor always cheats after failing to invest.

There are two cases, as visible on Supplementary Figure 3. When $k \geq c_L$, both of our thresholds are equal:

$$k \geq c_L \implies \delta_{2,s}^{\text{invest}} = \delta_{2,s}^{\text{recip.}} = \frac{k}{k+b-c_L}.$$

In other words, when the cost of investing is greater than or equal to the low cost of reciprocation, actor types can be divided into two: the actor does not invest and would cheat if trusted if $\delta < \frac{k}{k+b-c_L}$, and she invests and reciprocates if $\delta \geq \frac{k}{k+b-c_L}$.

The obtained strategy profile is cooperative if:

$$\mathbb{P}(\delta \geq \frac{k}{k+b-c_L}) > 0. \tag{3.3}$$

In that case, the reputation (invested, unknown) is attained with positive probability, and since every investor type also reciprocates, it is strictly beneficial for choosers to trust in that case. Analytically, we have:

$$k \geq c_L \implies \mathbb{P}(\delta \geq \delta_{2,s}^{\text{recip.}} \mid \delta \geq \delta_{2,s}^{\text{invest}}) = 1,$$

which means that condition (3.5) is automatically verified.

The reputation (invested, good) is also attained with positive probability, and it is immediately strictly beneficial for choosers to trust given that reputation.

The reputation (invested, bad) is impossible. We can assign beliefs to choosers in this case. Since the actor cheats after investing only when $\delta < \frac{c_L}{b}$, such types must exist with positive probability; otherwise, it would strictly beneficial for choosers to deviate to trusting in this case. In other words, we must have:

$$\mathbb{P}(\delta < \frac{c_L}{b}) > 0. \tag{3.4}$$

Finally, since the actor cheats after not investing regardless of her type, it is always strictly beneficial for choosers to distrust given any of the last three reputations (regardless of whether their beliefs are inferred or assigned).

Putting everything together, when $k \geq c_L$, we have shown that the first signaling equilibrium is a SPE iff conditions (3.3) and (3.4) are verified, condition (3.5) being automatically true.

Continuing with the first signaling equilibrium, we consider the other parameter case, $k < c_L$. Investing is then easier than reciprocation:

$$k < c_L \implies \delta_{2,s}^{\text{invest}} = \frac{k}{b} < \frac{c_L}{b} = \delta_{2,s}^{\text{recip.}}.$$

Actors can then be divided into three rather than two types: those that do not invest and cheat ($\delta < \frac{k}{b}$), those that invest and reciprocate ($\delta \geq \frac{c_L}{b}$) and, for intermediate discount factors ($\frac{k}{b} \leq \delta < \frac{c_L}{b}$), those that invest but cheat.

The obtained strategy profile is cooperative if:

$$\mathbb{P}(\delta \geq \frac{c_L}{b}) > 0, \tag{3.3}$$

in which case, the actor invests and reciprocates with positive probability.

It is then strictly beneficial for choosers to trust given (invested, good). In contrast to before, it is not automatically beneficial for choosers to trust given (invested, unknown), because there can exist types that invest and cheat. By trusting in this case, a chooser incurs cost γ and receives β with probability $\mathbb{P}(\delta \geq \frac{c_L}{b} \mid \delta \geq \frac{k}{b})$. This conditional probability is always defined; it is equal to 1 if intermediate types do not exist, and otherwise strictly smaller than 1.

Since not trusting yields null payoff, we deduce that there is no beneficial deviation for choosers when the actor invested iff:

$$\mathbb{P}(\delta \geq \frac{c_L}{b} \mid \delta \geq \frac{k}{b}) \geq \frac{\gamma}{\beta}. \tag{3.5}$$

In contrast to before, the reputation (invested, bad) can be possible: it is attained if there exist intermediate types, and is otherwise impossible. In both cases, it is necessary and sufficient to have:

$$\mathbb{P}(\delta < \frac{c_L}{b}) > 0. \tag{3.4}$$

This condition is implied by the existence of intermediate types, in which case it choosers' form beliefs through Bayesian inference that justify distrust, and otherwise allows us to assign them beliefs to justify distrust.

As before, it is always strictly beneficial for choosers to distrust given any of the last three reputations, occuring after the actor opts not to invest.

Putting everything together, when $k < c_L$, we have shown that the first signaling equilibrium is a SPE iff conditions (3.3-3.5) are verified, proving Proposition 3.2.

To prove Proposition 3.3, we move on to the second signaling equilibrium. The only difference is that choosers now trust given (didn't invest, good), and that non-investors, who are not trusted along the outcome path, would reciprocate if and only if their discount factor satisfies $\delta \geq \frac{c_H}{b}$.

There are now three parameter cases, as visible on Figure 3. The third case is obtained when $k > \frac{c_H(b-c_L)}{b-c_H}$, in which case, even though our thresholds are equal ($k > \frac{c_H(b-c_L)}{b-c_H} \implies k \geq c_L$), and actor types can be divided into two depending on their behavior along the outcome path, they can be divided into four in terms of strategy: very impatient types, who do not invest and always cheat ($\delta < \frac{c_L}{b}$), impatient types, who do not invest but would reciprocate if they had ($\frac{c_L}{b} \leq \delta < \frac{c_H}{b}$), patient types, who do not invest and always reciprocate ($\frac{c_H}{b} \leq \delta < \frac{k}{k+b-c_L}$), and very patient types, who invest and reciprocate ($\delta \geq \frac{k}{k+b-c_L}$).

The strategy profile is cooperative if:

$$\mathbb{P}(\delta \geq \frac{k}{k+b-c_L}) > 0, \tag{3.3}$$

guaranteeing the existence of very patient types. Under that condition, (invested, unknown) and (invested, good) are reached, and it is strictly beneficial to trust given both these reputations.

The reputation (invested, bad) is impossible. As with the first signaling equilibrium, to assign beliefs justifying distrust in this case, we need:

$$\mathbb{P}(\delta < \frac{c_L}{b}) > 0. \tag{3.4}$$

Distrust given the reputation (didn't invest, unknown) is no longer automatically beneficial: it depends on the proportion of patient types compared to the proportion of very impatient and impatient types. Comparing to the payoff of not trusting, we deduce that there is no beneficial deviation from distrusting if the actor did not invest iff:

$$\mathbb{P}(\delta \geq \frac{c_H}{b} \mid \delta < \frac{k}{k+b-c_L}) < \frac{\gamma}{\beta}, \tag{3.7}$$

the conditional probability being defined when condition (3.4) is verified, guaranteeing at least the existence of very impatient types.

Distrust given (didn't invest, bad) is strictly beneficial, whether because impatient types exist, making this reputation possible, or because we assign beliefs corresponding to very impatient types.

Finally, trust given (didn't invest, good) is strictly beneficial, whether because patient types exist, making this reputation possible, or because we assign beliefs corresponding to very patient types.

Putting everything together, when $k > \frac{c_H(b-c_L)}{b-c_H}$, we have shown that the second signaling equilibrium is an SPE iff condition (3.3), (3.4) and (3.7) are verified, conditions (3.5) and (3.6) being automatically true.

The other two parameter cases are similar to those for the first signaling equilibrium, the only difference being that condition (3.6) is necessary to justify trust given (didn't invest, good).

In both cases, the strategy profile is cooperative if and only if condition (3.3) is verified. This allows us to define the conditional probability in condition (3.5), which is necessary and sufficient for trust given (invested, unknown). It also makes trust given (invested, good) strictly beneficial.

In addition, distrust given (invested, bad) requires condition (3.4). This condition guarantees the existence of types who would cheat after investing, and therefore also of types who would cheat after not investing, making it strictly beneficial to distrust given (didn't invest, bad).

The only remaining reputation is (didn't invest, unknown). In both parameter cases, we have $\delta_{2,s}^{\text{invest}} \leq \frac{c_H}{b}$, meaning that the first part of condition (3.7) is verified, and that it is strictly beneficial to distrust given this reputation. Specifically, either this reputation occurs and it is strictly beneficial to distrust (because every non-investor cheats), or every actor initially opts out of investing, which requires $k < c_L$ and $\mathbb{P}(\delta < \frac{k}{b}) = 0$, in which case we can assign beliefs corresponding to types that satisfy $\delta < \frac{c_H}{b}$, who must exist following condition (3.4).

□

C.2 Universal trust

We study here the universal trust equilibrium, in which choosers trust the actor if she invested and if she didn't invest.

Universal trust and the two previously studied signaling equilibria are the only cooperative equilibria: an equilibrium in which choosers distrust regardless of investment behavior cannot be cooperative, and a strategy profile in which choosers distrust if the actor invested and trust if she did not invest cannot be part of a SPE (or even of a Nash equilibrium), since not investing is then always strictly better for the actor, regardless of her discount factor (she earns $-k$ if she invests and at least δb if she does not invest).

Restatement of Proposition 3.4: Universal trust

In the universal trust equilibrium, choosers trust the actor when her reputation is (invested, unknown), (invested, good), (didn't invest, unknown), or (didn't invest, good), and distrust her given (invested, bad) or (didn't invest, bad). The actor invests if and only if her discount factor δ satisfies:

$$\delta \geq \delta_{2,u}^{\text{invest}} \equiv \frac{k}{k + c_H - c_L}. \tag{3.8}$$

After investing, the actor reciprocates if and only if her discount factor satisfies $\delta \geq \frac{c_L}{b}$; after not investing, she reciprocates if and only if $\delta \geq \frac{c_H}{b}$. Overall, the actor reciprocates if and only if her discount factor satisfies:

$$\delta \geq \delta_{2,u}^{\text{recip.}} \equiv \frac{c_H}{b}. \tag{3.9}$$

This equilibrium exists if and only if:

$$0 < \mathbb{P}(\delta \geq \delta_{2,u}^{\text{invest}}), \tag{3.10}$$

$$0 < \mathbb{P}(\delta_{2,u}^{\text{recip.}} \leq \delta < \delta_{2,u}^{\text{invest}}), \tag{3.11}$$

$$0 < \mathbb{P}(\delta < \frac{c_L}{b}), \tag{3.12}$$

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \delta_{2,u}^{\text{recip.}} \mid \delta < \delta_{2,u}^{\text{invest}}). \tag{3.13}$$

Proof. Consider a PBE in which choosers trust both investors and non-investors. The probability that the actor will reciprocate must be positive in both cases — otherwise it would be beneficial for choosers to deviate to distrusting. It follows that the actor must be incentivized to reciprocate in both subgames: choosers must trust given (invested, good) and (didn't invest, good), and distrust given (invested, bad) and (didn't invest, bad).

As a result, the actor faces the same incentive to invest as in the cooperative equilibrium of the first extension: choosers will trust her in round 1 regardless of her investment decision, and subsequently, she will be trusted as long as she reciprocates.

We deduce that the actor invests and reciprocates under the same conditions, and that her strategy can be represented as in Supplementary Figure 2. In addition, since there must exist types that do not invest but reciprocate, we deduce that we must be in the case of large k represented on the right of this figure, i.e., that $\frac{k}{k+c_H-c_L} > \frac{c_H}{b}$, and that we must have:

$$\mathbb{P}(\frac{c_H}{b} \leq \delta < \frac{k}{k + c_H - c_L}) > 0. \tag{3.11}$$

To guarantee the existence of types that invest and reciprocate, we must also have:

$$\mathbb{P}(\delta \geq \frac{k}{k + c_H - c_L}) > 0. \tag{3.10}$$

It is then strictly beneficial to trust given both (invested, good) and (didn't invest, good), both of which are possible. For compatibility with distrust given (invested, bad) and (didn't invest, bad), it is necessary and sufficient to have:

$$\mathbb{P}(\delta < \frac{c_L}{b}) > 0. \tag{3.12}$$

Finally, both (invested, unknown) and (didn't invest, unknown) are attained. In the first case, it is strictly beneficial to trust, because every investor also reciprocates. In the second case, it isn't beneficial for choosers to deviate to distrusting if and only if:

$$\frac{\gamma}{\beta} \leq \mathbb{P}(\delta \geq \frac{c_H}{b} \mid \delta < \frac{k}{k + c_H - c_L}). \tag{3.13}$$

□